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JPL PUBLICATION 78-2

A Highly Accurate Method for the Determination of Mass and Center of Mass of a Spacecraft

(NASA-CR-156130) A HIGHLY ACCURATE METHOD FOR THE DETERMINATION OF MASS AND CENTER OF MASS OF A SPACECRAFT (Jet Propulsion Lab.) 63 p HC A04/MF A01 CSCL 22B

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National Aeronautics and
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California Institute of Technology
Pasadena, California

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16. Abstract

An extremely accurate method for the measurement of mass and the lateral center of mass of a spacecraft has been developed. The method was needed for the Voyager spacecraft mission requirement which limited the uncertainty in the knowledge of lateral center of mass of the spacecraft system weighing 750 kg to be less than 1.0 mm (0.04 in.).

The method consists of using three load cells symmetrically located at 120° apart on a turntable with respect to the vertical axis of the spacecraft and making six measurements for each load cell. These six measurements are taken by cyclic rotations of the load cell turntable and of the spacecraft, about the vertical axis of the measurement fixture. This method eleiminate all alignment, leveling, and load cell calibration errors for the ateral center of mass determiniation, and permits a statistical best fit of the measurement data. An associate data reduction computer program called MASCM has been written to implement this method and has been used for the Voyager spacecraft. The accuracy obtained was 0.14 mm (0.006 in.) for the center of mass and o.01% for the mass determinations.

The method and its associated data reduction computer program have been so designed that they are suitable for any other mass and center of mass determination.

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A Highly Accurate Method for the Determination of Mass and Center of Mass of a Spacecraft

E. Y. Chow
M. R. Trubert
A. Egwuatu

April 15, 1978

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
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PREFACE

The work described in this report was performed by the Applied Mechanics Division of the Jet Propulsion Laboratory.

ACKNOWLEDGMENT

The authors wish to acknowledge the pioneering effort of the late Mr. William H. Gayman, whose early fundamental thoughts and analyses led to the successful development of the measurement and data reduction techniques for obtaining a highly accurate mass and center of mass prediction of the Voyager Spacecraft.

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ABSTRACT

An extremely accurate method for the measurement of mass and the lateral center of mass of a spacecraft has been developed. The method was needed for the Voyager spacecraft mission requirement which limited the uncertainty in the knowledge of lateral center of mass of the spacecraft system weighing 750 kg to be less than 1.0 mm (0.04 in.).

The method consists of using three load cells symmetrically located at 120° apart on a turntable with respect to the vertical axis of the spacecraft and making six measurements for each load cell. These six measurements are taken by cyclic rotations of the load cell turntable and of the spacecraft, about the vertical axis of the measurement fixture. This method eliminates all alignment, leveling, and load cell calibration errors for the lateral center of mass determination, and permits a statistical best fit of the measurement data. An associated data reduction computer program called MASCM has been written to implement this method and has been used for the Voyager spacecraft. The accuracy obtained was 0.14 mm (0.006 in.) for the center of mass and 0.01% for the mass determinations.

The method and its associated data reduction computer program have been so designed that they are suitable for any other mass and center of mass determinations.

1. SUMMARY

An extremely accurate method for the measurement of mass and center of mass of a spacecraft is reported here. The method consists of using three load cells symmetrically located 120° apart on a turntable with respect to the vertical axis of the spacecraft and making six measurements for each load cell. These six measurements are taken by cyclic rotations of the turntable and one rotation of the spacecraft about its vertical axis. This method eliminates all alignment, leveling and load cells calibration errors for measurement of the center of mass, and permits a statistical best fit of the data.

A data reduction computer program called MASCM has been written to implement the method and has been used on the 750 kg Voyager spacecraft. The accuracy (standard deviation) obtained was 0.14 mm for the center of mass and 0.01% for the mass.

2. METHOD

The spacecraft is supported at the apices, 1, 2, and 3 of an equilateral triangle through an adapter ring and a load plate as shown in Figs. 1 and 2. Three load cells connect these three points to a common turntable which in turn rests on the ground. Two steps representing six sets of measurements are needed to eliminate alignment, spacecraft tiltings and load cell calibration errors. Details of the method are found in Appendix F.

2.1 Center of Mass Measurement

Step 1. 120° turntable rotations -- Three sets of measurements are taken with the load cell turntable assembly rotated 120° each time with respect to the spacecraft assembly and ground. Using measurement data for these three positions eliminates the load cell calibration errors (Appendix F) and the

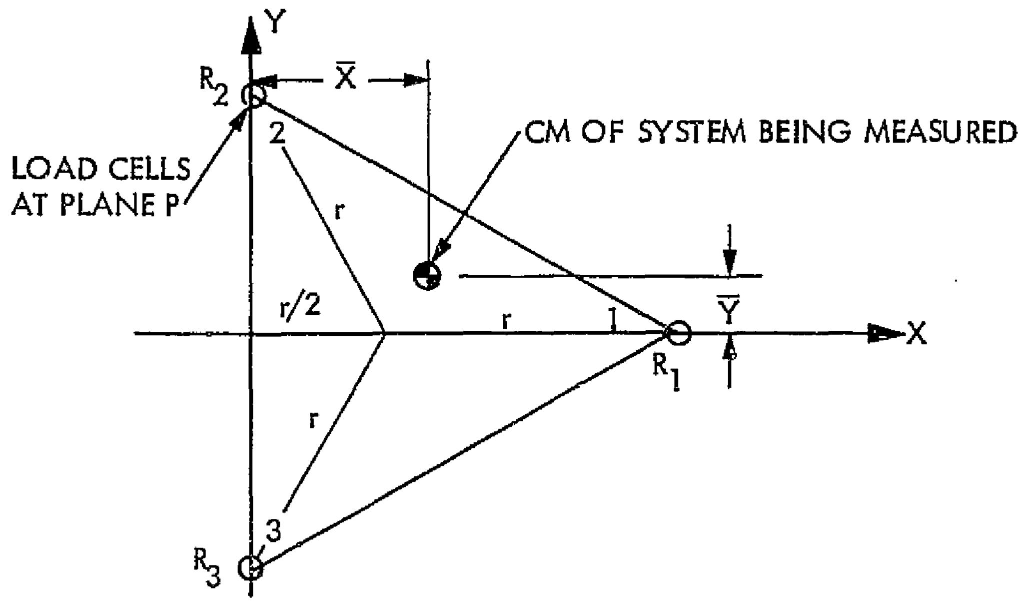

Fig. 1 System Geometry and Notation

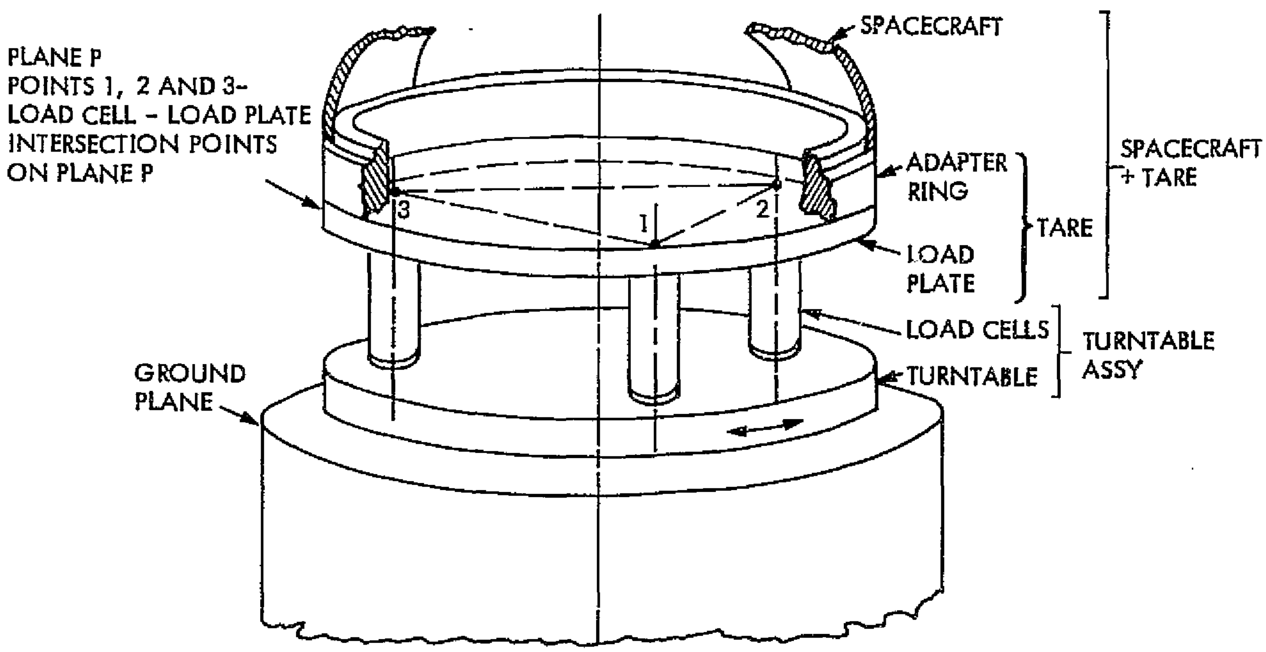

Fig. 2. General Arrangement of Mass and Center of Mass Measurement Fixture

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nonparallelism of top and bottom planes of the load cell turntable, see angle β of Fig. 3 in the center of mass calculation. The load cell readings R_{ij} are different for each of the three positions, and generate nine readings tabulated as follows:

Load Cells \ Measurements	1	2	3
1	R_{11}	R_{12}	R_{13}
2	R_{22}	R_{23}	R_{21}
3	R_{33}	R_{31}	R_{32}

where R_{ij} - load cell reading; i - load cell identification number and

j - load cell location relative to spacecraft points 1, 2, 3.

For example, R_{32} is the reading of load cell No. 3 at location 2 on the spacecraft. The coordinates $\bar{x}_j$, $\bar{y}_j$ of the projection of the center of mass of the spacecraft assembly, including tare, on the interface plane P between the load plate and the load cell (Fig. 3) are calculated by (see Appendix F):

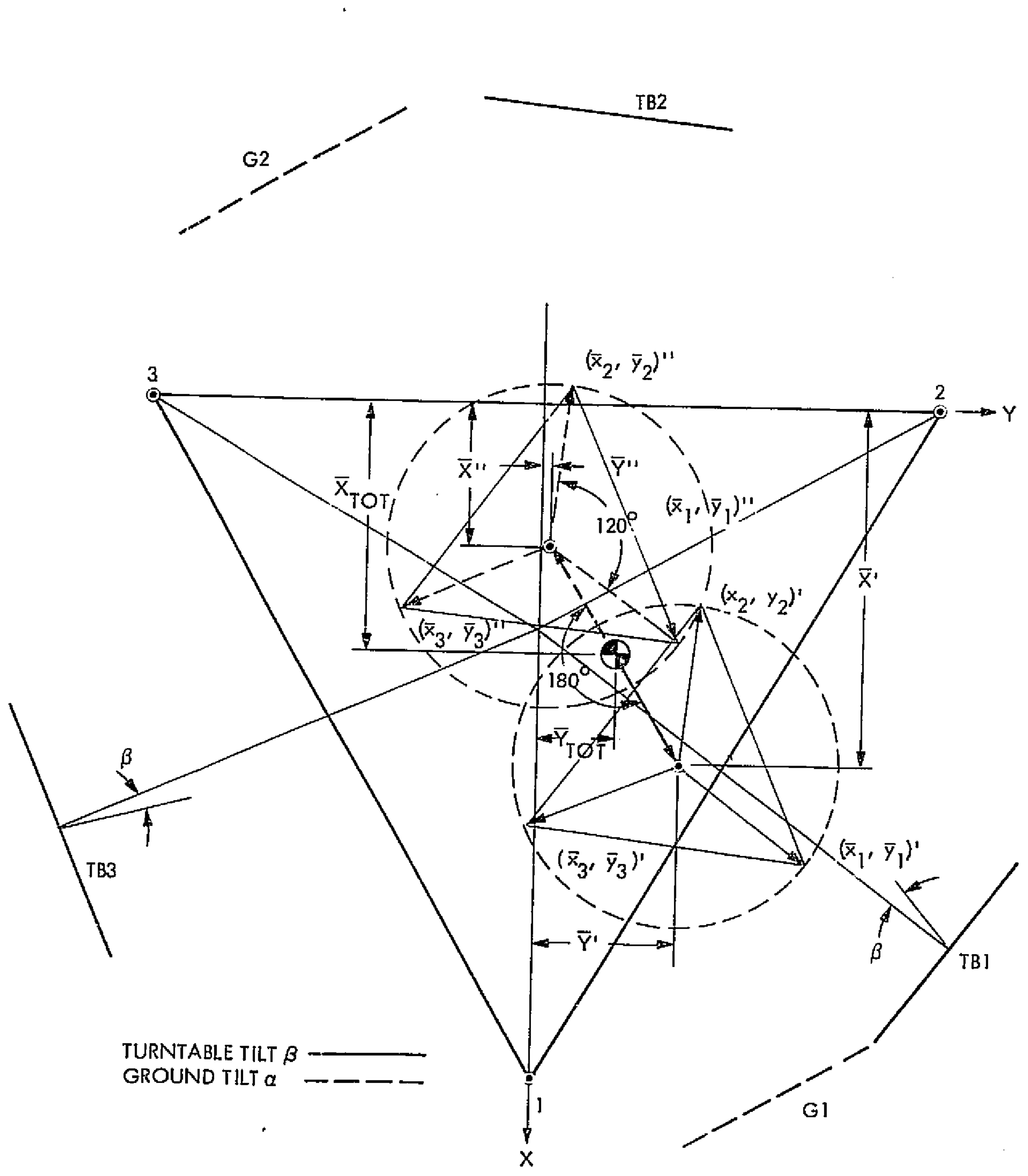
$$\bar{x}'_1 = \frac{3}{2} r \frac{R_{11}}{W_1}; \quad \bar{y}'_1 = \frac{r\sqrt{3}}{2} \frac{R_{22} - R_{33}}{W_1} \tag{1}$$

$$\bar{x}'_2 = \frac{3}{2} r \frac{R_{31}}{W_2}; \quad \bar{y}'_2 = \frac{r\sqrt{3}}{2} \frac{R_{12} - R_{23}}{W_2} \tag{2}$$

$$\bar{x}'_3 = \frac{3}{2} r \frac{R_{21}}{W_3}; \quad \bar{y}'_3 = \frac{r\sqrt{3}}{2} \frac{R_{32} - R_{13}}{W_3} \tag{3}$$

where

$$W_1 = R_{11} + R_{22} + R_{33}, \; W_2 = R_{12} + R_{23} + R_{31}, \text{ and } W_3 = R_{13} + R_{21} + R_{32} \tag{4}$$



$(\bar{X}_{TOT}, \bar{Y}_{TOT})$: ACTUAL CM

$(\bar{X}', \bar{Y}')$ AND $(\bar{X}'', \bar{Y}'')$: APPARENT CM DUE TO α TILT ANGLE

$(\bar{x}_i, \bar{y}_i)'$ AND $(\bar{x}_i, \bar{y}_i)''$: APPARENT CM DUE TO β TILT ANGLE

Fig. 3. Projection of Centers of Mass on P-Plane

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are the masses for each position, nominally different from each other because of calibration errors. Then, the coordinates $\bar{X}'$ and $\bar{Y}'$ of the projection of the center of mass are the mean value of the three sets of measurements.

$$\bar{X}' = \frac{1}{3} (\bar{x}_1' + \bar{x}_2' + \bar{x}_3') \text{ and } \bar{Y}' = \frac{1}{3} (\bar{y}_1' + \bar{y}_2' + \bar{y}_3') \tag{5}$$

Step 2. <u>180° Spacecraft rotation</u> -- Measurements are taken for the spacecraft rotated 180° with respect to the load base plate held fixed relative to the ground. The turntable is then again rotated three times as before to obtain the second set of data.

Rotating the spacecraft 180° eliminates the ground plane tilt error (angle α), load plate and fixture ring and concentricity errors (see Appendix C). Using the same equations as above, a second set of coordinates $\bar{X}''$ and $\bar{Y}''$ is obtained. Then the coordinates $\bar{X}_{TOT}$ and $\bar{Y}_{TOT}$ of the center of mass of the spacecraft with tare are:

$$\bar{X}_{TOT} = \frac{1}{2} (\bar{X}' + \bar{X}'') \text{ and } \bar{Y}_{TOT} = \frac{1}{2} (\bar{Y}' + \bar{Y}'') \tag{6}$$

2.2 Mass Measurement

The mass of the spacecraft with tare W_{TOT} is the average value of the mass for each set

$$W_{TOT} = \frac{1}{6} \sum_{i,j} \lambda_i R_{ij} \tag{7}$$

where λ_i are the load cell calibration factors.

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2.3 Tare

The center of mass and mass of the tare must be removed from $\bar{X}_{TOT}$, $\bar{Y}_{TOT}$ and W_{TOT}. Since the tare has a well-defined geometry, the coordinates $\bar{X}_T$ and $\bar{Y}_T$ of the tare center of mass can be calculated with good accuracy. Hence a measurement as described above for the tare is not necessary but can be performed if desired. Also, since the tare is much lighter than the spacecraft, $\bar{X}_T$ and $\bar{Y}_T$ have only a small influence on the result.

2.4 Net Spacecraft

The center of mass coordinates $\bar{X}_{S/C}$ and $\bar{Y}_{S/C}$ and the mass $W_{S/C}$ of the spacecraft are computed by

$$\bar{X}_{S/C} = \frac{W_{TOT}}{W_{S/C}} \bar{X}_{TOT} \tag{8}$$

$$\bar{Y}_{S/C} = \frac{W_{TOT}}{W_{S/C}} \bar{Y}_{TOT} \tag{9}$$

$$W_{S/C} = W_{TOT} - W_T \tag{10}$$

where W_T is the mass of the tare.

2.5 Error Estimate

The following can be used to estimate an upper bound of the errors before and after the measurement.

a) Error on center of mass location of spacecraft plus tare

The errors not eliminated by load cell and spacecraft rotations are estimated from the formula of Appendix F to which an origin of coordinates error Δx_o, Δy_o is added.

$$\Delta\bar{X} \le \frac{\Delta r}{2} + \frac{2}{3} r \frac{\Delta P}{P} + \Delta x_o \tag{11}$$

$$\Delta\bar{Y} \le \frac{r\sqrt{3}}{3} \frac{\Delta P}{P} + \Delta y_o \tag{12}$$

Note that there is no error from the calibration factor because of the clocking of the load cells which measure the three reactions successively. The readout error ΔP is a random error made of three parts ΔP_1, ΔP_2 and ΔP_3 (Appendix H):

ΔP_1 is the readout drift error

ΔP_2 is the readout error due to resolution

ΔP_3 is the readout fluctuation due to environment.

b) Error on center of mass location of spacecraft

From Eqs. (8) and (9) the center of mass error are

$$\Delta\overline{X}_{S/C} = \frac{W_{TOT}}{W_{S/C}} \Delta\overline{X} \tag{13}$$

$$\Delta\overline{Y}_{S/C} = \frac{W_{TOT}}{W_{S/C}} \Delta\overline{Y} \tag{14}$$

where $\Delta\overline{X}$ and $\Delta\overline{Y}$ are results of Eqs. (11) and (12)

c) Error on mass of net spacecraft

The error on the mass of the spacecraft is calculated by equation of ΔP of Appendix H

$$\Delta W = \frac{\Delta\lambda}{\lambda} W + 3\ \Delta P$$

where the load cell calibration accuracy $\frac{\Delta\lambda}{\lambda}$ is now introduced with the $\Delta\lambda$ being the error on the calibration factor λ.

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2.6 Statistical Errors

The center of mass error estimates calculated in Section 2.5 are only bounds. After obtaining the measurement data, a best fit solution to the points $(x_1,y_1)'$, $(x_2,y_2)'$, ... in the XY plane as shown in Fig. 3 is obtained. The best fit criteria is that of parallel equal equilateral triangles and permit the determination of the standard deviation which is an indication of the quality of the measurements. Appendix G shows the derivation of the statistical best fit scheme and the resulting standard derivation.

2.7 Load Cell Calibration and Requirements

Accurate load cell calibrations, as discussed in Appendix F, are needed only to obtain an accurate measurement of spacecraft mass but are not required for the center of mass. The ranges of needed accuracy for the Voyager spacecraft measurement are from 45 to 90 kg and from 275 to 365 kg. Sample calibration curves are shown in Appendix D.

The following were the instrumentation requirements to meet the Voyager spacecraft measurement accuracy limits:

1.	Load cell range	1000 kg
2.	Readout resolution	.05 kg
3.	Load cell readout system calibration accuracy (in 45-90 kg and 275-365 kg range)	.1%
4.	Load cell location accuracy	.05 mm

3. Data Reduction Computer Program MASCM

3.1 Program Summary

The computer program MASCM provides the means of computations of the mass and the center of mass of the spacecraft from data obtained by the measurement operations as described in Section 2, and the estimation of the standard derivations defined by Eqs. (2) through (9) of Appendix G. The usage of MASCM is described below. Relevant mathematical derivations and a listing of the program are in Section 2 and the Appendices.

The lateral center of mass location ($\bar{x}$, $\bar{y}$) output from MASCM is referred to the spacecraft coordinate system rather than to the load cell coordinate system used in Section 2. The coordinate transformation from the load cell to the spacecraft coordinate systems is described in Appendix B. In addition Appendix B shows the manner in which the program handles the cases of tare measurements and spacecraft plus tare measurements, the method of correction for these measurements and the calibration data. It is also shown that the lateral center of mass of the tare is not required for the calculation of the spacecraft lateral center of mass.

The measured mass and center of mass values of the Voyager spacecraft have been determined through the use of the MASCM program and are presented in Table 1. All uncertainties are 1σ (one sigma) value.

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TABLE 1. Mass and Center of Mass Prediction For The Voyager Spacecraft (Based on Measurement Data)

SPACECRAFT (PARTIAL)	MASS kg (lb)	$\bar{X}$ cm (in)	$\bar{Y}$ cm (in)	ΔCG cm (in)
(PTM) VGR77-2	706.266 ± 0.083 (1557.05 ± 0.183)	+1.191 ± 0.010 (+0.469 ± 0.004)	+2.332 ± 0.010 (+0.918 ± 0.004)	±0.014 (±0.006)
(S/C 31) VGR77-2	687.814 ± 0.091 (1516.37 ± 0.200)	-0.079 ± 0.034 (-0.031 ± 0.013)	+1.143 ± 0.034 (+0.450 ± 0.013)	±0.048 (±0.019)
(S/C 32) VGR77-3	709.350 ± 0.096 (1563.85 ± 0.212)	+0.132 ± 0.034 (+0.052 ± 0.013)	+3.625 ± 0.034 (+1.427 ± 0.013)	±0.048 (±0.019)

3.2 Program Description

The MASCM program is a computer program written in the MBASIC language. It is designed to compute the mass and the center of mass and their associated standard deviations from sets of data using Eqs. (2) through (9) of Appendix G. A listing of the program is shown in Appendix A.

The MASCM program is called either from the MBASIC file or from tape and is loaded into the computer for execution. It can be executed either from the demand terminal or through the batch processor. For a batch run the input data are punched on cards and the deck of cards submitted to the computer. For the demand terminal, the input data are entered stepwise into the computer as

the computer requests them (see Appendix E, Sample Problem). The order in which the data are punched on cards or entered on the demand terminal is the same for both batch and demand terminal modes.

The technique as described in Section 2 requires measurements to be performed with the rotation of the load cell turntable assembly to three different positions and with the spacecraft in each of the two positions, 0° and 180° rotation with respect to the tare. Therefore, two tables of load cell data for the spacecraft plus tare case representing 18 inputs are needed for inputs.

Because of design simplification of the spacecraft handling scheme during measurement, there is only one set of load cell accommodation holes on the load plate (see Fig. 2 of Section 2). Therefore, the load plate does not have the capability of being measured in the two positions and only one table of load cell data for the tare is obtained. The MASCM program is designed to have the capability of handling cases of either one table (as the tare case) or two tables (as the spacecraft plus tare case) of load cell data. The particular case desired for computation is implemented by exercising the option--Type 1 or Type 2 respectively--in the program.

The input can be either in the metric units or the U.S. customary units.

3.3 Program Usage

3.3.1 The input data to the program consists of:

- The test (measurement) identification number
- The radius, r of the load cell location, see Fig. 1 and Eqs. (3) through (6) of Appendix B
- The angle, θ of the load cell system orientation, see Fig. 1 and Eqs. (7) and (8) of Appendix B

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- The coordinates x_o, y_o of load cell system origin O_p, see Fig. 1 and Eqs. (11) and (12) of Appendix B.
- The load cell readout data, R_{ij} as tabulated in Section 2
- The configuration of the spacecraft that is undergoing measurement.
- The tare mass and CM data
- Option (Type 1 or Type 2) to select the case of one or two tables of load cell readings
- Option (1 or 2) to select input units (metric or U.S. customary).

3.3.2 <u>The key to the Input/Output Parameter:</u>

RADIUS: Average radius of load cell locations

MASS: Mass of a specified portion of spacecraft

XBAR: Abscissa of a CM location

YBAR: Ordinate of a CM location

SIGMA: Standard deviation of a CM location

SGMAW: Standard deviation of mass

SGMCAL: Standard deviation from the load cell calibration curves.

R1(I,J), R2(I,J) refer to the two sets of load cell readings corresponding to the 0° and 180° rotations of the spacecraft. Each set is as shown in Section 2. These values must be entered by row. An alphabet T following any of the above parameters indicates that the tare is being referred to, for example, XBART = abscissa of the center of mass location of the tare alone.

3.3.3. Sample Runstreams

The user may store this program on a magnetic tape. For example, for tape Y000, the runstream is as follows:

```
@RUN XXX,XXXXXX,XXXXX
@CAT,P FILE.
@ASG,A FILE.
@ASG,T T,T, Y000R
@REWIND T.
@CØPY,G T.,FILE.
@FREE T.
@MBASIC                    (Accesses MBASIC)
LOAD 'FILE.MASCM'          (Accesses MASCM program for execution)
RUN                        (Execution of MASCM program)
 |
 |
 |
 |
 |-INPUT DATA (demand terminal entries or batch input deck)
 |
 |
 └ØUTPUT PRINTØUT
 |
 |
EXIT
@DELETE FILE.
@FIN
```

The user may store this program in the MBASIC file under his password if he wishes to access the program from the MBASIC file for subsequent runs. In this case the runstream is as follows:

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```
@RUN XXX,XXXXXX,XXXXX (Run card)
@MBASIC XXXXXX        (Accesses MBASIC and its files)
LOAD 'MASCM'          (Accesses MASCM program for execution)
RUN                   (Execution of MASCM program)
 |
 |
 |
 |
 |— Input Data (demand terminal entries or batch input deck)
 |
 |
 |
 |
 |— Output Printout
 |
@FIN
```

The order of the input data is shown in the Appendix E (Sample Problem). This order is valid for both demand terminal and batch runs as discussed in the beginning of Section 3.2.

4. COORDINATE TRANSFORMATION

Define $\bar{x}_p$, $\bar{y}_p$ as the apparent center of mass of the spacecraft being measured in the load cell system of coordinates, then a corresponding center of mass $\bar{x}$, $\bar{y}$ in the spacecraft system is given by Eq. (10a) of Appendix B and repeated here for convenience.

$$\begin{Bmatrix} \bar{x} \\ \bar{y} \end{Bmatrix} = \begin{Bmatrix} x_o \\ y_o \end{Bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \bar{x}_p \\ \bar{y}_p \end{Bmatrix}$$

where x_o, y_o is the origin of load cell system with respect to spacecraft system and θ is angle of rotation of the spacecraft system with respect to the load cell system (Fig. 4).

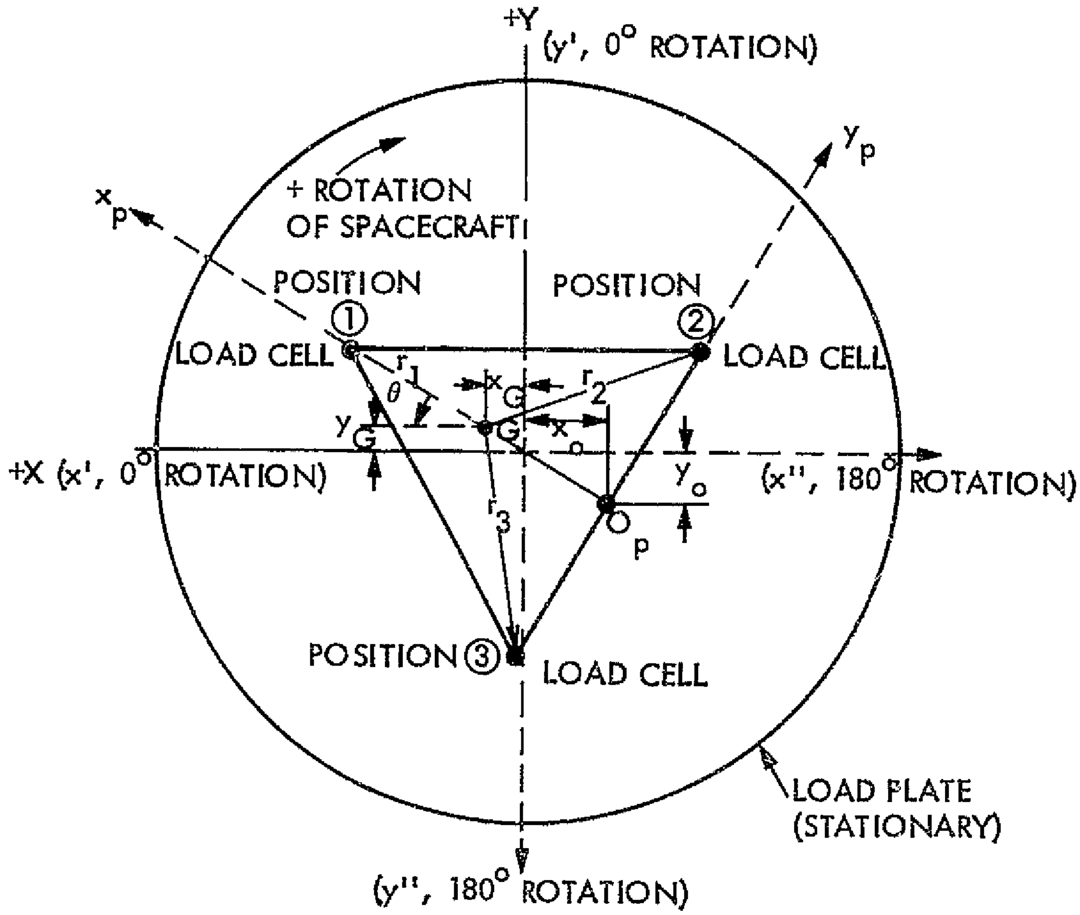

Fig. 4. Loadcell Geometry

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5. LOAD CELL CALIBRATION AND READOUT CORRECTION

All load cell data R_{ij} obtained during the mass and center of mass measurement of the spacecraft must be recorded in accordance with the requirement delineated in Section 2.1. The readouts R_{ij} must be corrected for the load cell calibration error before input to the program. The calibration error is obtained in Appendix D. An average value for each corrected R_{ij} must be generated if more than one complete set of nine measurements is performed since each complete set must contain only nine R_{ij}.

The value of SGMCAL (σ_{CAL}) used for standard deviation estimate in the program is the average value of the standard deviations of all three load cells throughout the applicable load range. It is established from data obtained by the load cell calibration test. Typical calibration data are presented in Appendix D.

6. REMOVAL OF TARE

As stated in Section 2 the load plate remains in the original position after the first set of measurements while the spacecraft is rotated 180° for the second set of measurements. To obtain the net spacecraft center of mass in the spacecraft coordinate system, the following mathematical expressions have been derived and are included in the MASCM program where W_T is the mass of the load plate plus the mass of the adapter ring (tare). The derivation is shown in Appendix C.

$$\bar{x}_{S/C} = \left(\frac{W_{S/C} + W_T}{W_{S/C}}\right)\bar{x}; \quad \bar{y}_{S/C} = \left(\frac{W_{S/C} + W_T}{W_{S/C}}\right)\bar{y}$$

$$\sigma_{S/C} = \left(\frac{W_{S/C} + W_T}{W_{S/C}}\right)\sigma; \quad \sigma_{W_{S/C}} = \sqrt{\sigma^2_{W_{S/C+T}} + \sigma^2_{W_T}}$$

Appendix A. PROGRAM LISTING

MASCM

```
2 PRINT 'INPUT TEST NO.'
3 INPUT TESTNO
5 PRINT 'INPUT 1 IF UNITS ARE METRIC; 2 IF UNITS ARE U.S. CUSTOMARY'
7 INPUT SYSTEM
9 REAL X(6),Y(6),SUMX(7),SUMY(7),A(3),B(3),D(4),D1(3),D2(3),D3(3),D4(3)
11 REAL R(3,3),R1(3,3),R2(3,3),W(3),W1(3),W2(3),XD(6),YD(6)
13 PRINT 'INPUT THE CONFIGURATION'
15 INPUT CONFG$
17 PRINT 'INPUT RADIUS,WEIGHTT,XBART,YBART,SGMAWT,SIGMAT,SGMCAL'
21 INPUT SR,WT,XBART,YBART,SGMAWT,SGMAT,SGMCAL
23 PRINT 'INPUT X0,Y0,THETA IN DG.'
25 INPUT X0,Y0,THETA
27 PRINT 'INPUT R1(I,J)  ROW BY ROW'
29 INPUT R1(1,1),R1(1,2),R1(1,3),R1(2,2),R1(2,3),R1(2,1),R1(3,3),R1(3,1),&
R1(3,2)
30 PRINT' INPUT 1 OR 2 FOR ONE OR TWO TABLES OF  L/C  READINGS'
31 INPUT MESUD
32 XBART2=XBART
33 YBART2=YBART
34 SGMAT2=SGMAT
35 IF MESUD=1 THEN GO TO 42
36 XBART=0.
38 YBART=0.
40 SGMAT=0.
42 R=R1
44 GOSUB 500
46 W1=W
48 COSTH=COS(THETA)
50 SINTH=SIN(THETA)
51 X(I)=X0+COSTH*XD(I)+SINTH*YD(I) FOR I=1 TO 3
52 Y(I)=Y0-SINTH*XD(I)+COSTH*YD(I) FOR I=1 TO 3
53 IF MESUD=1 THEN GO TO 62
54 PRINT 'INPUT  R2(I,J)  ROW BY ROW'
55 INPUT R2(1,1),R2(1,2),R2(1,3),R2(2,2),R2(2,3),R2(2,1),R2(3,3),&
R2(3,1),R2(3,2)
56 R=R2
57 GOSUB 500
60 X(I+3)=-X0-COSTH*XD(I)-SINTH*YD(I) FOR I=1 TO 3
61 Y(I+3)=-Y0+SINTH*XD(I)-COSTH*YD(I) FOR I=1 TO 3
62 W2=W
63 IF MESUD=2 THEN GO TO 67
65 X(I+3)=X(I) FOR I=1 TO 3
66 Y(I+3)=Y(I) FOR I=1 TO 3
67 SUMX(1)=0,SUMY(1)=0
70 FOR I=1 TO 6
73 SUMX(I+1)=SUMX(I)+X(I)
75 SUMY(I+1)=SUMY(I)+Y(I)
80 NEXT I
85 XBAR=SUMX(7)/6
90 YBAR=SUMY(7)/6
95 ALFA=(2*SUMX(4)-SUMX(7))/6
100 BETA=(2*SUMY(4)-SUMY(7))/6
110 BB=(3*Y(1)+3*Y(4)-SUMY(7)-SQR(3)*(X(2)-X(3)+X(5)-X(6)))/12
120 AA=(3*X(1)+3*X(4)-SUMX(7)+SQR(3)*(Y(2)-Y(3)+Y(5)-Y(6)))/12
130 A(1)=AA
132 A(2)=-(AA+SQR(3)*BB)/2
```


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134 A(3)=-(AA-SQR(3)*BB)/2
136 B(1)=BB
138 B(2)=(SQR(3)*AA-BB)/2
140 B(3)=-(SQR(3)*AA+BB)/2
143 D(1)=0, WEIGHT=0
145 FOR I=1 TO 3
147 WEIGHT=WEIGHT+(W1(I)+W2(I))/6
150 D1(I)=X(I)-(XBAR+ALFA+A(I))
162 D2(I)=Y(I)-(YBAR+BETA+B(I))
164 D3(I)=X(I+3)-(XBAR-ALFA+A(I))
166 D4(I)=Y(I+3)-(YBAR-BETA+B(I))
190 D(I+1)=D1(I)**2+D2(I)**2+D3(I)**2+D4(I)**2+D(I)
200 NEXT I
210 SIGMA=SQR(D(4)/5)
212 SGMWSQ=0
213 FOR I=1 TO 3
215 SGMWSQ=(WEIGHT-W1(I))**2+(WEIGHT-W2(I))**2+SGMWSQ
217 NEXT I
220 SGMAW=SQR(SGMWSQ/5+SGMCAL**2)
240 WSC=WEIGHT-WT
250 XBARSC=WEIGHT*XBAR/WSC-WT*XBART/WSC
260 YBARSC=WEIGHT*YBAR/WSC-WT*YBART/WSC
270 SGMASC=SQR((WEIGHT*SIGMA/WSC)**2+(WT*SGMAT/WSC)**2)
275 SGMWSC=SQR(SGMAW**2+SGMAWT**2)
276 PRINT\\\
277 PRINT USING'                              TEST  NO. %%%':TESTNO
278 PRINT\
280 IF SYSTEM=1 THEN GO TO 284
281 PRINT '        1. MEASURED  C.G.  AND  WEIGHT'
282 GO TO 285
284 PRINT '        1. MEASURED  C.M.  AND  MASS'
285 PRINT\
286 PRINT USING '          CONFIGURATION:  #':CONFG$
287 PRINT\
288 PRINT'          INPUT'
289 PRINT
290 IF SYSTEM=1 THEN GO TO 295
292 PRINT '        WEIGHT READINGS (LB)'
294 GO TO 298
295 PRINT '        MASS READINGS (KG)'
298 FMT5$='(/3(3B3%.3%))'
300 FMT1$='(/2(3(3B3%.3%)6B))'
302 IF MESUD=2 THEN GO TO 310
305 PRINT USING FMT5$: R1(1,1),R1(1,2),R1(1,3),&
R1(2,2),R1(2,3),R1(2,1),R1(3,3),R1(3,1),R1(3,2)
308 IF MESUD=1 THEN GO TO 313
310 PRINT\'          CASE 1.                                CASE 2.'
312 PRINT USING FMT1$:&
R1(1,1),R1(1,2),R1(1,3),R2(1,1),R2(1,2),R2(1,3),&
R1(2,2),R1(2,3),R1(2,1),R2(2,2),R2(2,3),R2(2,1),&
R1(3,3),R1(3,1),R1(3,2),R2(3,3),R2(3,1),R2(3,2)
313 FMT2$='<(9B'XBAR (IN):'5%.4%/9B'YBAR (IN):'5%.4%/8B'SIGMA (IN):'&
5%.4%/)>'
314 FMT22$='<(9B'XBAR (CM):'5%.4%/9B'YBAR (CM):'5%.4%/8B'SIGMA (CM):'&
5%.4%/)>'
316 FMT7$='<(9B'XBAR (CM):'5%.4%16B'XBAR (IN):'5%.4%/9B'YBAR (CM):'&
5%.4%16B'YBAR (IN):'5%.4%)>'
```

```
317 FMT77$='<(/8B'SIGMA (CM):'5%.4%15B'SIGMA (IN):'5%.4%)>'
318 FMT3$='<(7B'WEIGHT (LB):'5%.4%/8B'SGMAW (LB):'5%.4%/)>'
319 FMT33$='<(7B'  MASS (KG):'5%.4%/8B'SGMAW (KG):'5%.4%/)>'
320 FMT8$='<(9B'XBAR (IN):'5%.4%16B'XBAR (CM):'5%.4%/9B'YBAR (IN):'&
5%.4%16B'YBAR (CM):'5%.4%/)>'
321 FMT88$='<(/8B'SIGMA (IN):'5%.4%15B'SIGMA (CM):'5%.4%)>'
324 FMT4$='<(7B'SGMCAL (LB):'5%.4%/)>'
325 FMT6$='<(7B'SGMCAL (KG):'5%.4%/)>'
326 PRINT\
327 IF SYSTEM=1 THEN GO TO 332
328 PRINT USING'       RADIUS (IN):   %%.%%%%':SR
329 PRINT USING FMT4$:SGMCAL
330 GO TO 336
332 PRINT USING'       RADIUS (CM):   %%.%%%%':SR
334 PRINT USING FMT6$:SGMCAL
336 PRINT
338 GOSUB 600
340 PRINT'           OUTPUT'
342 PRINT
344 IF SYSTEM=1 THEN GO TO 353
345 PRINT USING FMT8$:XBAR,XBAR2,YBAR,YBAR2
346 PRINT USING FMT88$:SIGMA,SIGMA2
347 FMT9$='<(/7B'WEIGHT (LB):'5%.4%15B' MASS (KG):'5%.4%)>'
348 FMT99$='<(/8B'SGMAW (LB):'5%.4%15B'SGMAW (KG):'5%.4%/)>'
350 PRINT USING FMT9$:WEIGHT,WT2
351 PRINT USING FMT99$:SGMAW,MAW2
352 GO TO 359
353 PRINT USING FMT7$:XBAR,XBAR2,YBAR,YBAR2
354 PRINT USING FMT77$:SIGMA,SIGMA2
355 FMT10$='<(/8B' MASS (KG):'5%.4%14B'WEIGHT (LB):'5%.4%)>'
356 FMT11$='<(/8B'SGMAW (KG):'5%.4%15B'SGMAW (LB):'5%.4%)>'
357 PRINT USING FMT10$:WEIGHT,WT2
358 PRINT USING FMT11$:SGMAW,MAW2
359 PRINT\\
360 PRINT'       2. TARE'
362 PRINT\
370 PRINT '          INPUT'
372 PRINT
376 IF SYSTEM=1 THEN GO TO 386
380 PRINT USING FMT2$:XBART2,YBART2,SGMAT2
382 PRINT USING FMT3$:WT,SGMAWT
384 GO TO 390
386 PRINT USING FMT22$:XBART2,YBART2,SGMAT2
388 PRINT USING FMT33$:WT,SGMAWT
390 PRINT\\
395 IF SYSTEM=1 THEN GO TO 405
400 PRINT USING'       3.  MEASURED  #  C.G. AND WEIGHT':CONFG$
402 GO TO 410
405 PRINT USING'       3.  MEASURED  #  C.M. AND MASS':CONFG$
410 PRINT
415 PRINT '          OUTPUT'
420 PRINT
425 IF SYSTEM=1 THEN GO TO 438
430 PRINT USING FMT8$:XBARSC,XSC2,YBARSC,YSC2
431 PRINT USING FMT88$:SGMASC,SGSC2
434 PRINT USING FMT9$:WSC,WSC2
435 PRINT USING FMT99$:SGMWSC,SMWSC2
```

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436 GO TO 480
438 PRINT USING FMT7$:XBARSC,XSC2,YBARSC,YSC2
439 PRINT USING FMT77$:SGMASC,SGSC2
440 PRINT USING FMT10$:WSC,WSC2
442 PRINT USING FMT11$:SGMWSC,SMWSC2
480 END
500 W(1)=R(1,1)+R(2,2)+R(3,3),  W(2)=R(1,2)+R(2,3)+R(3,1)
510 W(3)=R(1,3)+R(2,1)+R(3,2)
520 V=3*SR/2,  U=SQR(3)*SR/2
530 XD(1)=V*R(1,1)/W(1),  XD(2)=V*R(3,1)/W(2),  XD(3)=V*R(2,1)/W(3)
540 YD(1)=U*(R(2,2)-R(3,3))/W(1), YD(2)=U*(R(1,2)-R(2,3))/W(2)
550 YD(3)=U*(R(3,2)-R(1,3))/W(3)
575 RETURN
600 IF SYSTEM=2 THEN GO TO 630
602 XBAR2=XBAR*.3937
604 YBAR2=YBAR*.3937
606 SIGMA2=SIGMA*.3937
608 XSC2=XBARSC*.3937
610 YSC2=YBARSC*.3937
612 SGSC2=SGMASC*.3937
614 WT2=WEIGHT*2.204622
616 MAW2=SGMAW*2.204622
618 WSC2=WSC*2.204622
620 SMWSC2=SGMWSC*2.204622
625 GO TO 650
630 XBAR2=XBAR/.3937
632 YBAR2=YBAR/.3937
634 SIGMA2=SIGMA/.3937
636 XSC2=XBARSC/.3937
638 YSC2=YBARSC/.3937
640 SGSC2=SGMASC/.3937
642 WT2=WEIGHT/2.204622
644 MAW2=SGMAW/2.204622
646 WSC2=WSC/2.204622
648 SMWSC2=SGMWSC/2.204622
650 RETURN
```

Appendix B. COORDINATE TRANSFORMATION

Figure B.1 shows the position or location of the load cells on the fixture load plate (tare). Points 1, 2, and 3 are the load cell locations which, nominally, form an equilateral triangle. The coordinates of the vertices referred to spacecraft system are indicated on the figure. For the convenience of describing the coordinate transformation, numerical expressions corresponding to Voyager have been included. Point G is the centroid of the triangle and its coordinates (x_G, y_G) are:

$$x_G = (x_1 + x_2 + x_3)/3 = 0.00630 \text{ in.} \tag{1}$$

$$y_G = (y_1 + y_2 + y_3)/3 = 0.00267 \text{ in.} \tag{2}$$

where $(x_1 y_1)$, $(x_2 y_2)$ and $(x_3 y_3)$ are the coordinates of points 1, 2 and 3 respectively.

The calculated radii between G and points 1, 2 and 3 are:

$$r_1 = \left[(x_1 - x_G)^2 + (y_1 - y_G)^2 \right]^{1/2} = 9.49879 \text{ in.} \tag{3}$$

$$r_2 = \left[(x_2 - x_G)^2 + (y_2 - y_G)^2 \right]^{1/2} = 9.49908 \text{ in.} \tag{4}$$

$$r_3 = \left[(x_3 - x_G)^2 + (y_3 - y_G)^2 \right]^{1/2} = 9.49917 \text{ in.} \tag{5}$$

These three radii would be identical and have a design value of 9.500 in. if triangle 1 2 3 was a perfect equilateral triangle. However, due to the

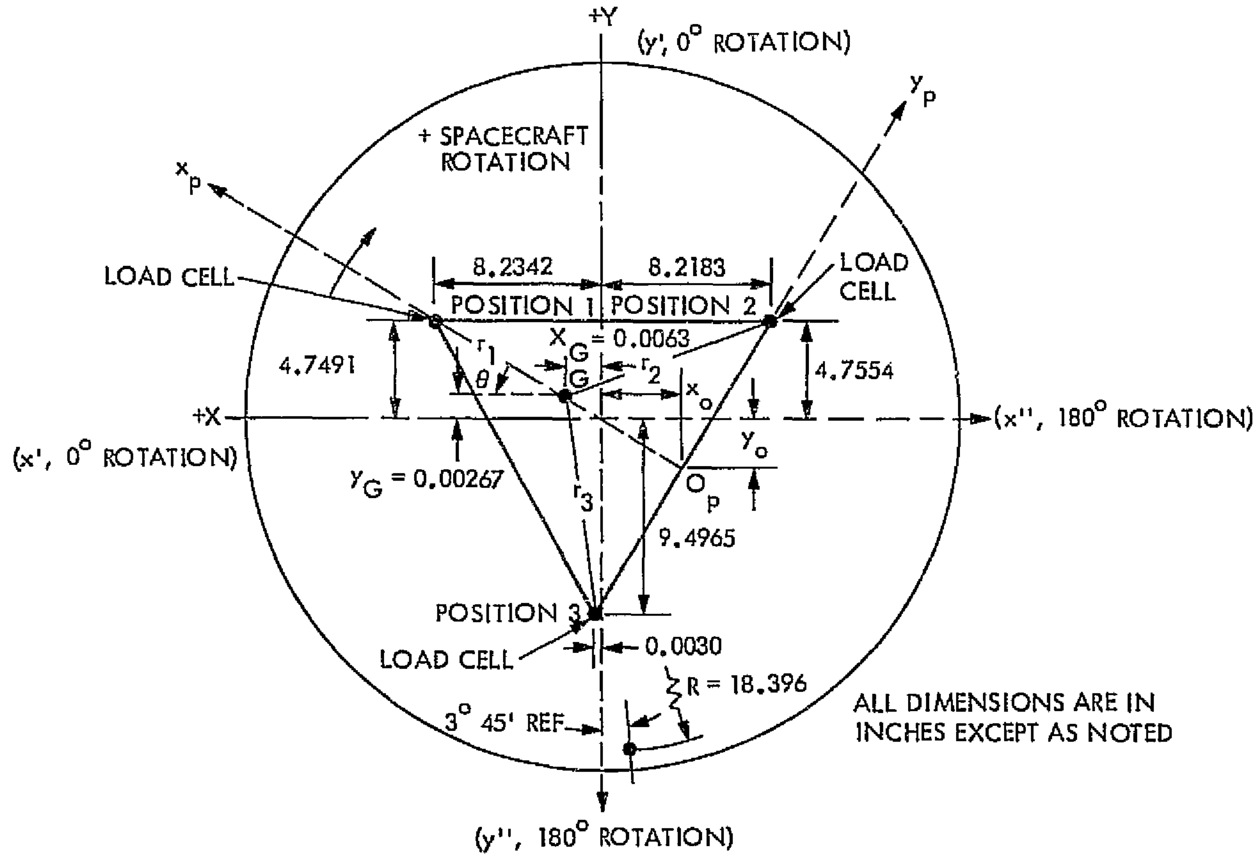

Fig. B.1. Actual Dimensions of the Voyager Mass and CM Measurement Fixture Load Plate

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manufacturing tolerance, it is only approximately so and the radius r is taken as the average of the three radii

$$r = \frac{r_1 + r_2 + r_3}{3} = 9.49901 \text{ in.} \tag{6}$$

Note that triangle ① ② ③ is equilateral within .001 in.

Call x_p, y_p the load cell coordinate system with its origin at 0_p and θ the angle of rotation of x, y system with respect to x_p, y_p system, then:

$$\sin \theta = \frac{y_1 - y_G}{r} = -\frac{4.791 - 0.00267}{9.49901} = -0.499673 \text{ in.} \tag{7}$$

$$\cos \theta = \frac{x_1 - x_G}{r} = \frac{8.2342 - 0.00630}{9.49901} = 0.866214 \text{ in.} \tag{8}$$

Eqs. (7) and (8) imply that $\theta = -29.9784^\circ$.

Fig. B.2 depicts the coordinate transformation for any point $Q(x_p, y_p)$ in x_p, y_p system to Q(x,y) in the x,y system.

$$\therefore x = \overline{OF} = \overline{OD} + \overline{DE} + \overline{EF}$$

and

$$y = \overline{OC} = \overline{OA} + \overline{AB} + \overline{BC}$$

i.e.,

$$x = x_o + x_p \cos \theta + y_p \sin \theta \tag{9}$$

and

$$y = y_o + y_p \cos \theta - x_p \sin \theta \tag{10}$$

where x_o, y_o are the coordinates of 0_p relative to x,y system.

Rewriting Eqs. (9) and (10) in matrix form, we have

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} x_o \\ y_o \end{Bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} x_p \\ y_p \end{Bmatrix} \tag{10a}$$

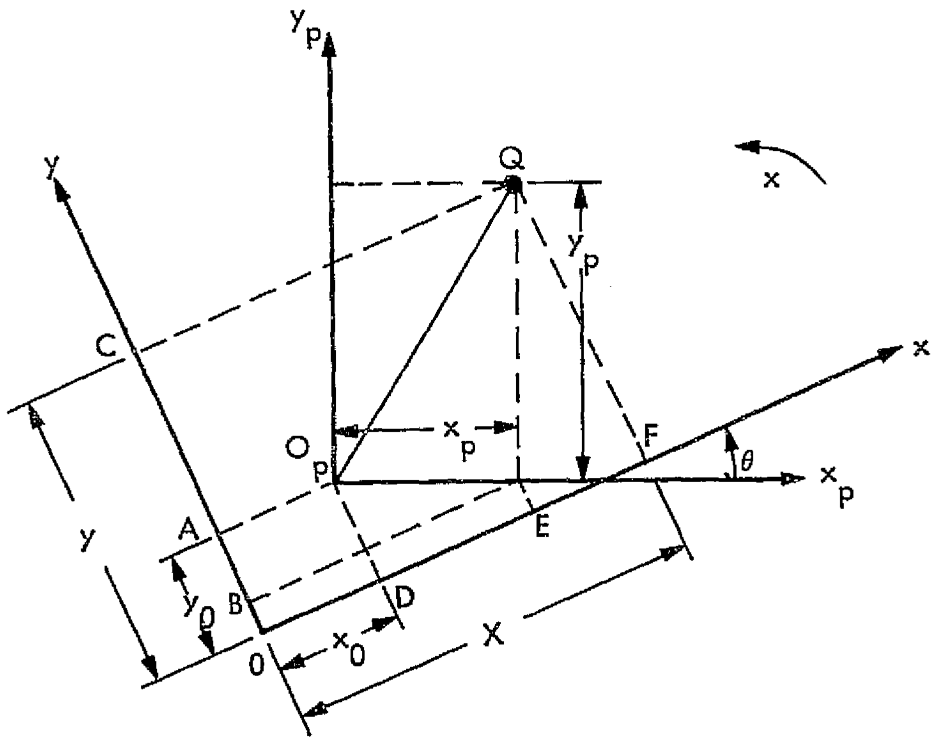

Fig. B.2. Coordinate Transformation

Eq. (10a) is now written for the Voyager geometry. The coordinates of 0_p referred to a system parallel to x_p, y_p with origin at G is $(-\overline{GO}_p, 0)$. By using Eqs. (9) and (10), the coordinates of point 0_p with respect to the x, y system are given as:

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$$x_o = (-\overline{GO}_p) \cos \theta + x_G = -4.107788 \text{ in.} \tag{11}$$

$$y_o = (-\overline{GO}_p) \sin \theta + y_G = -2.370499 \text{ in.} \tag{12}$$

where $\overline{GO}_p = \frac{1}{2} r$

Substituting the numerical values of x_o, y_o and θ and calling x', y' the coordinate corresponding to the 0^o position of the spacecraft we obtain the transformation

$$x' = -4.107788 + 0.866214 \, (x_p) - 0.499673 \, (y_p) \tag{13}$$

$$y' = -2.370499 + 0.499673 \, (x_p) + 0.866214 \, (y_p) \tag{14}$$

For the second set of measurements the spacecraft with the adapter ring was rotated 180^o with respect to the load plate or tare (See Sec. 2). Recalling that the spacecraft coordinate system is fixed with the spacecraft, the appropriate transformation, corresponding to the 180^o position of the spacecraft, is obtained from Eqs. (13) and (14) by a change of sign.

$$x'' = 4.107788 - 0.866214 \, (x_p) + 0.499673 \, (y_p) \tag{15}$$

$$y'' = 2.370499 - 0.499673 \, (x_p) - 0.866214 \, (y_p) \tag{16}$$

Eqs. (13) through (16) are the transformation equations based on load cell geometry as shown in Fig. B.1.

Appendix C. REMOVAL OF THE TARE

From Fig. C-1 the spacecraft is at 0^{o} position relative to the tare, then,

$$W_T \bar{x}_T + W_{S/C} (a + \bar{x}_{S/C}) \cos \alpha = (W_{S/C} + W_T) \bar{x}' \cos \alpha \qquad (1)$$

where

$\bar{x}'$ = combined spacecraft and tare C.G.

α = nonparallelism of the top of load plate and the referenced horizontal ground plane.

From Fig. C-2 the spacecraft plus the attached adapter ring only is at 180^{o} position relative to the tare, then,

$$-W_T \bar{x}_T - W_{S/C} (a - \bar{x}_{S/C}) \cos \alpha = (W_{S/C} + W_T) \bar{x}'' \cos \alpha \qquad (2)$$

where

$\bar{x}''$ = combined spacecraft and tare C.G.

From Eq. (1) and (2) above

$$2W_{S/C} \bar{x}_{S/C} \cos \alpha = (W_{S/C} + W_T) (\bar{x}' + \bar{x}'') \cos \alpha$$

$$\bar{x}_{S/C} = \frac{W_{S/C} + W_T}{W_{S/C}} \left(\frac{\bar{x}' + \bar{x}''}{2} \right)$$

$$\therefore \quad \bar{x}_{S/C} = \frac{W_{S/C} + W_T}{W_{S/C}} \bar{x} \qquad (3)$$

Similarly

$$\bar{y}_{S/C} = \frac{W_{S/C} + W_T}{W_{S/C}} \bar{y} \qquad (4)$$

where $\bar{x}$, and $\bar{y}$ are the C.G. of spacecraft plus tare and are not a function of α.

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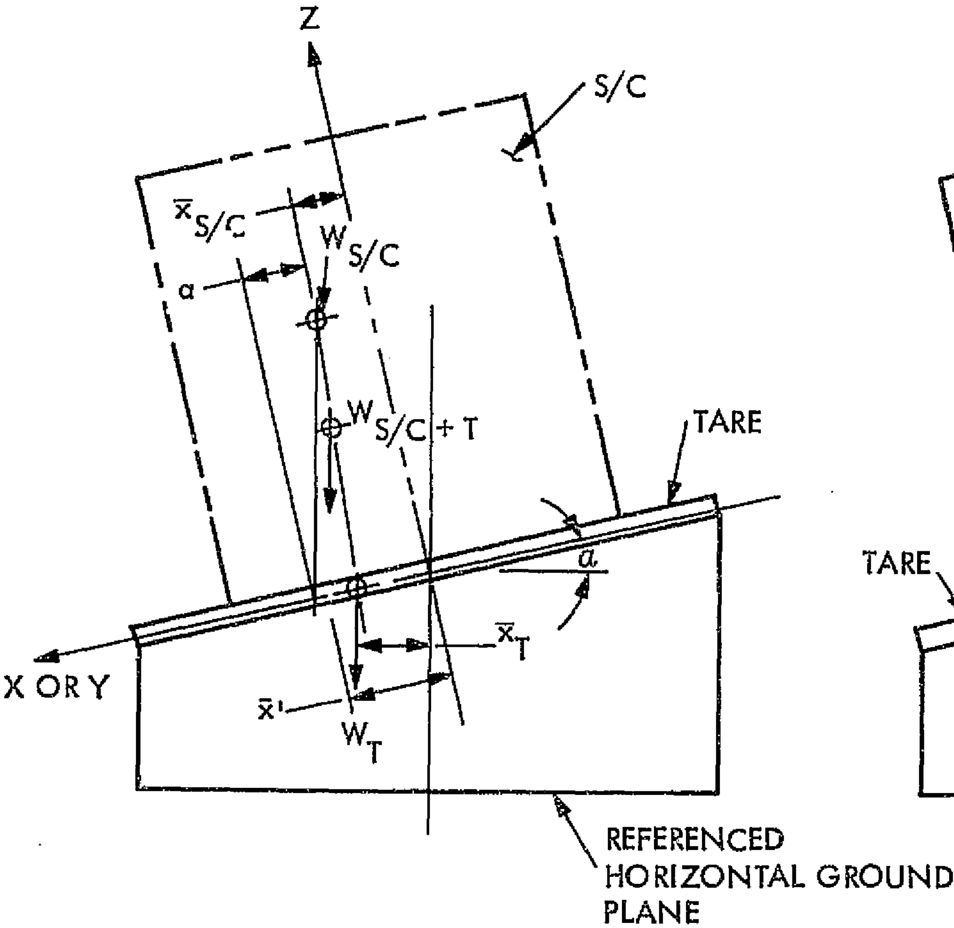

Fig. C.1. Spacecraft at 0°

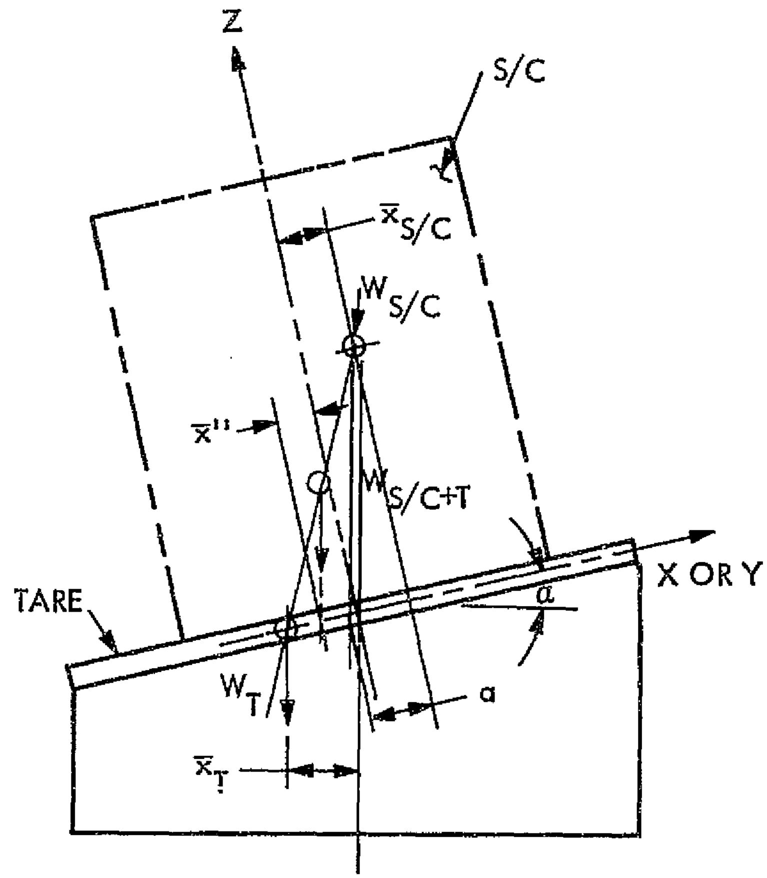

Fig. C.2. Spacecraft at 180°

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Eqs. (3) and (4) show that the C.G. coordinates of the tare are not required in the calculation of the spacecraft's C.G. Only the weight of the tare W_T is a required parameter. Then the $\sigma_{S/C}$ is in the form of

$$\sigma_{S/C} = \left(\frac{W_{S/C} + W_T}{W_{S/C}}\right)\sigma \tag{5}$$

where σ is calculated standard deviation on the spacecraft plus tare C.G. coordinates based on input data. However, the standard deviation on the spacecraft weight estimate remain the same as that shown in Eq. (9) of Appendix G.

$$\sigma_{W_{S/C}} = \left[\sigma^2_{W_{S/C+T}} + \sigma^2_{W_T}\right]^{1/2} \tag{6}$$

where $\sigma_{W_{S/C+T}}$ and σ_{W_T} are calculated standard deviation on the weight of the spacecraft plus tare, and of the tare, respectively.

As depicted in Fig. 2 of Section 2, the tare for Voyager comprised the adapter ring plus the load plate with a total weight of about 140 kg (310 lbs). The field joint is located at the interface of these two parts. The spacecraft is detached from the load plate at the field joint for the 180° rotation with the adapter ring attached to it. Since first, the C.G. of the adapter ring is very close to the vertical axis of the spacecraft within 0.0025 cm (0.001 in.) and second, the weight of the ring is very small as compared to that of the spacecraft (1%), the ring can be included in the load plate tare weight W_T in Eqs. (3) and (4).

Appendix D. LOAD CELL CALIBRATION CURVES

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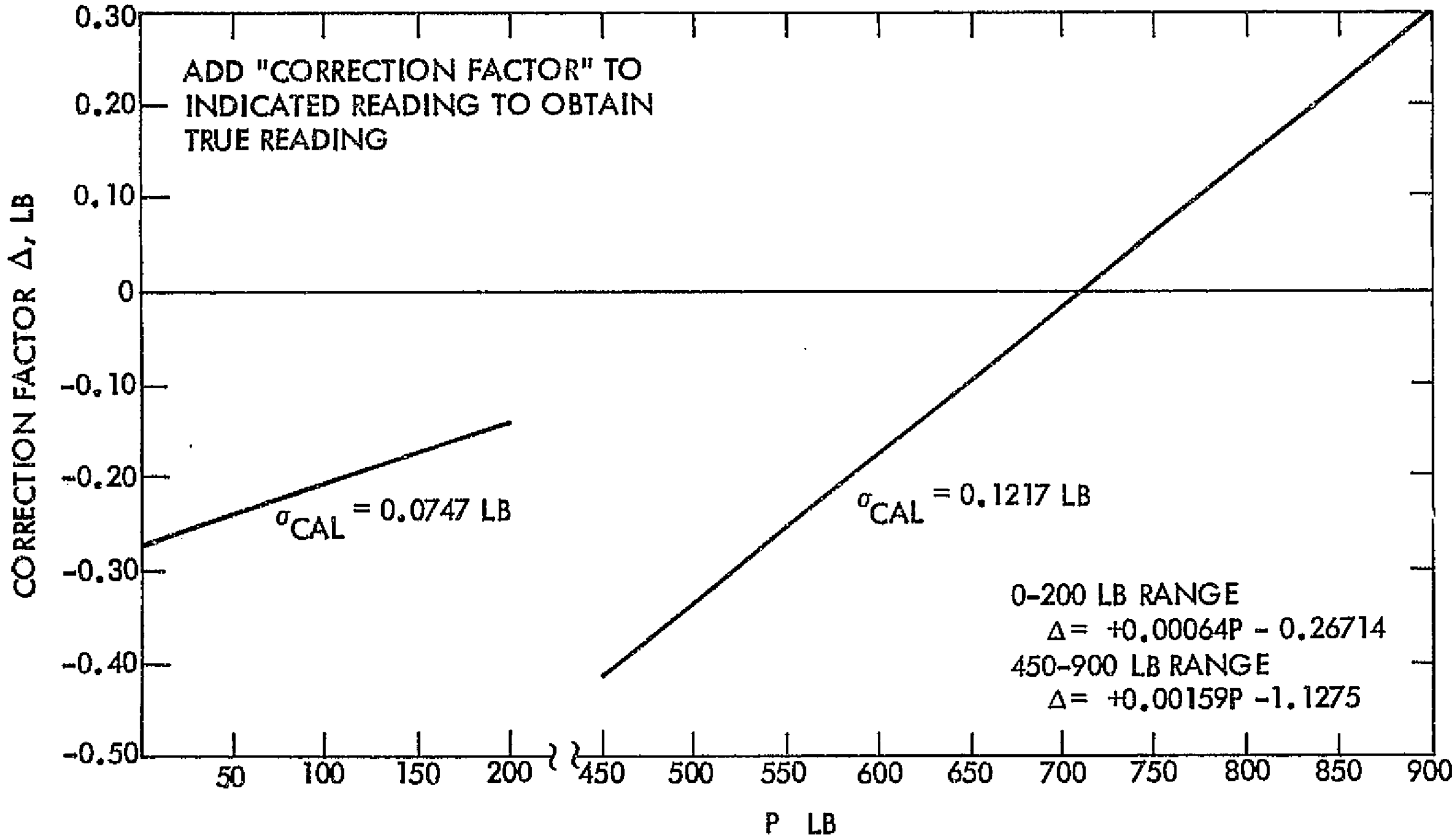

Fig. D.1. Calibration Curve, Load Cell No. 1 (S/N 460), Yellow
(Calibrated 1/30/76 and 2/2/76)

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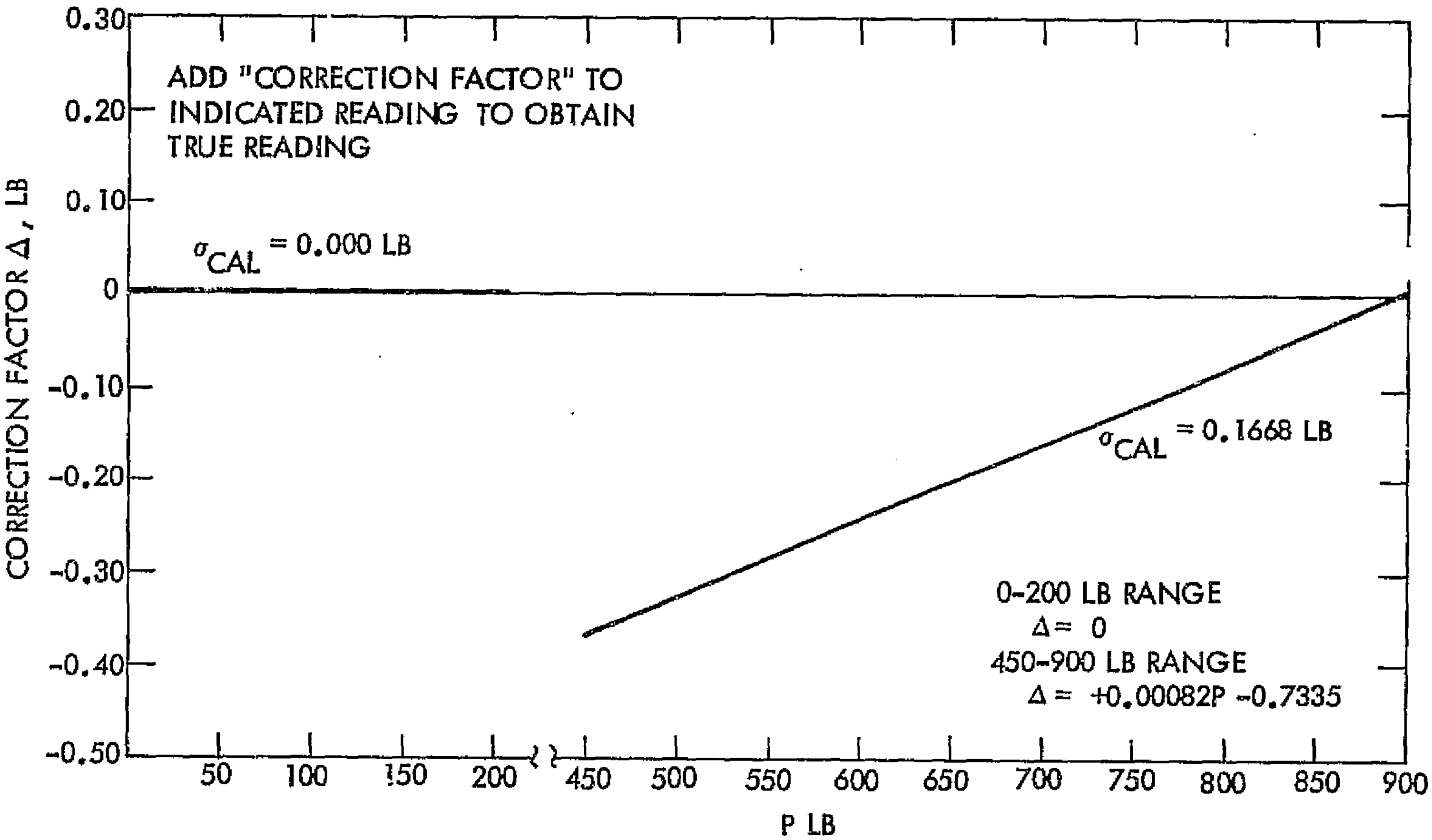

Fig. D.2. Calibration Curve, Load Cell No. 2 (S/N 461), Blue (Calibrated 3/17/76)

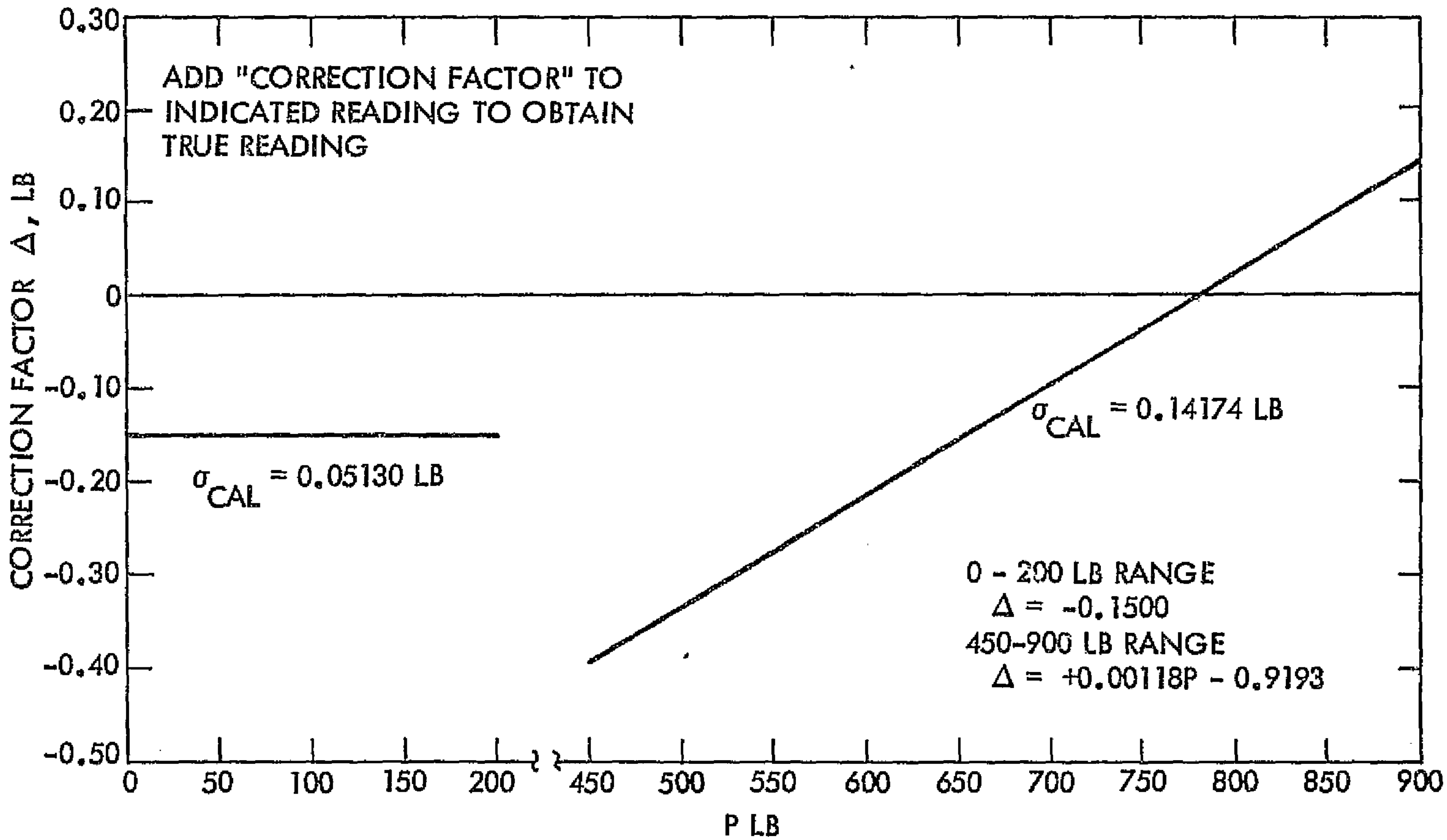

Fig. D.3. Calibration Curve, Load Cell No. 3 (S/N 462), Red
(Calibrated 2/3/76 and 2/4/76)

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```
INPUT TEST NO.
?1
INPUT 1 IF UNITS ARE METRIC; 2 IF UNITS ARE U.S. CUSTOMARY
?1
INPUT THE CONFIGURATION
?VGR-77  TARE ONLY
INPUT RADIUS,WEIGHTT,XBART,YBART,SGMAWT,SIGMAT,SGMCAL
?24.127,0.,0.,0.,0.,0.,.0191
INPUT X0,Y0,THETA IN DG.
?-10.433802,-6.021080,-29.9784
INPUT R1(I,J)  ROW BY ROW
?37.900,37.934,37.979,37.830,37.881,37.762,37.988,37.866,37.943
 INPUT 1 OR 2 FOR ONE OR TWO TABLES OF  L/C  READINGS
?1
```

```
                         TEST  NO.   1

      1. MEASURED  C.M.  AND  MASS

         CONFIGURATION:  VGR-77  TARE ONLY

         INPUT

      MASS READINGS  (KG)

   37.900    37.934    37.979
   37.830    37.881    37.762
   37.988    37.866    37.943

      RADIUS  (CM):   24.1270
      SGMCAL  (KG):     .0191

         OUTPUT

        XBAR  (CM):       .0048          XBAR  (IN):      .0019
        YBAR  (CM):     -.0096           YBAR  (IN):    -.0038
       SIGMA  (CM):       .0035         SIGMA  (IN):      .0014
        MASS  (KG):   113.6943         WEIGHT  (LB):  250.6530
       SGMAW  (KG):       .0265         SGMAW  (LB):      .0584
```

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2. TARE

INPUT

XBAR (CM):	0.0000
YBAR (CM):	0.0000
SIGMA (CM):	0.0000
MASS (KG):	0.0000
SGMAW (KG):	0.0000

3. MEASURED VGR-77 TARE ONLY C.M. AND MASS

OUTPUT

XBAR (CM):	.0048	XBAR (IN):	.0019
YBAR (CM):	-.0096	YBAR (IN):	-.0038
SIGMA (CM):	.0035	SIGMA (IN):	.0014
MASS (KG):	113.6943	WEIGHT (LB):	250.6530
SGMAW (KG):	.0265	SGMAW (LB):	.0584

```
>INPUT TEST NO.
?2
INPUT 1 IF UNITS ARE METRIC; 2 IF UNITS ARE U.S. CUSTOMARY
?1
INPUT THE CONFIGURATION
?VGR-2 PARTIAL S/C DRY W/O MIRIS
INPUT RADIUS,WEIGHTT,XBART,YBART,SGMAWT,SIGMAT,SGMCAL
?24.1270,113.6943,.0048,-.0096,.0265,.0035,.0650
INPUT X0,Y0,THETA IN DG.
?-10.433802,-6.021080,-29.9784
INPUT R1(I,J)  ROW BY ROW
?279.505,277.727,244.869,276.698,243.813,278.577,245.354,280.008,277.965
 INPUT 1 OR 2 FOR ONE OR TWO TABLES OF  L/C  READINGS
?2
INPUT  R2(I,J)  ROW BY ROW
?260.248,253.286,288.449,252.334,287.392,259.430,288.956,260.867,253.609
```

TEST NO. 2

1. MEASURED C.M. AND MASS

CONFIGURATION: VGR-2 PARTIAL S/C DRY W/O MIRIS

INPUT

MASS READINGS (KG)

CASE 1.			CASE 2.		
279.505	277.727	244.869	260.248	253.286	288.449
276.698	243.813	278.577	252.334	287.392	259.430
245.354	280.008	277.965	288.956	260.867	253.609

RADIUS (CM): 24.1270
SGMCAL (KG): .0650

OUTPUT

XBAR (CM):	-.0675	XBAR (IN):	-.0266
YBAR (CM):	.9839	YBAR (IN):	.3874
SIGMA (CM):	.0417	SIGMA (IN):	.0164
MASS (KG):	801.5145	WEIGHT (LB):	1767.0365
SGMAW (KG):	.0860	SGMAW (LB):	.1895

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2. TARE

INPUT

XBAR (CM):	.0048
YBAR (CM):	-.0096
SIGMA (CM):	.0035
MASS (KG):	113.6943
SGMAW (KG):	.0265

3. MEASURED VGR-2 PARTIAL S/C DRY W/O MIRIS C.M. AND MASS

OUTPUT

XBAR (CM):	-.0786	XBAR (IN):	-.0310
YBAR (CM):	1.1466	YBAR (IN):	.4514
SIGMA (CM):	.0485	SIGMA (IN):	.0191
MASS (KG):	687.8202	WEIGHT (LB):	1516.3835
SGMAW (KG):	.0900	SGMAW (LB):	.1983

Appendix F. DERIVATION OF THE MEASUREMENT METHOD

There is an advantage in using only one load cell successively to measure C.G. location. If a single load cell is employed, successively, in three known positions of a statically determinate fixture, with inert structural elements in the other two cells, the accuracy of the determination does not require an accurate knowledge of load cell calibration, but rather requires only that the load cell readout be linear with the applied load. An extension of this concept to a system incorporating three load cells and three successive sets of measurements in which each load cell senses each of the three reactions gives an improved accuracy.

Load cell calibrations are needed only to obtain a measurement of spacecraft weight, not for the determination of the center of gravity.

1. Equilibrium

Fig. F.1 defines the configuration, the notation, and the sign conventions chosen for this analysis.

At the outset, it is assumed that the manufacturing capabilities can provide a load cell placement accuracy that removes planform geometry from consideration as a significant contributor to the error. Thus the principal concern relates to errors associated with load cell readouts. Assume that the reactions due to the weight of the spacecraft are R_1, R_2 and R_3. The weight and C.G. platform axes are designated as X' and Y'. For the present, it is assumed that the spacecraft Z axis coincides with the platform Z' axis through the centroid of the equilateral triangle defined by the reaction points.

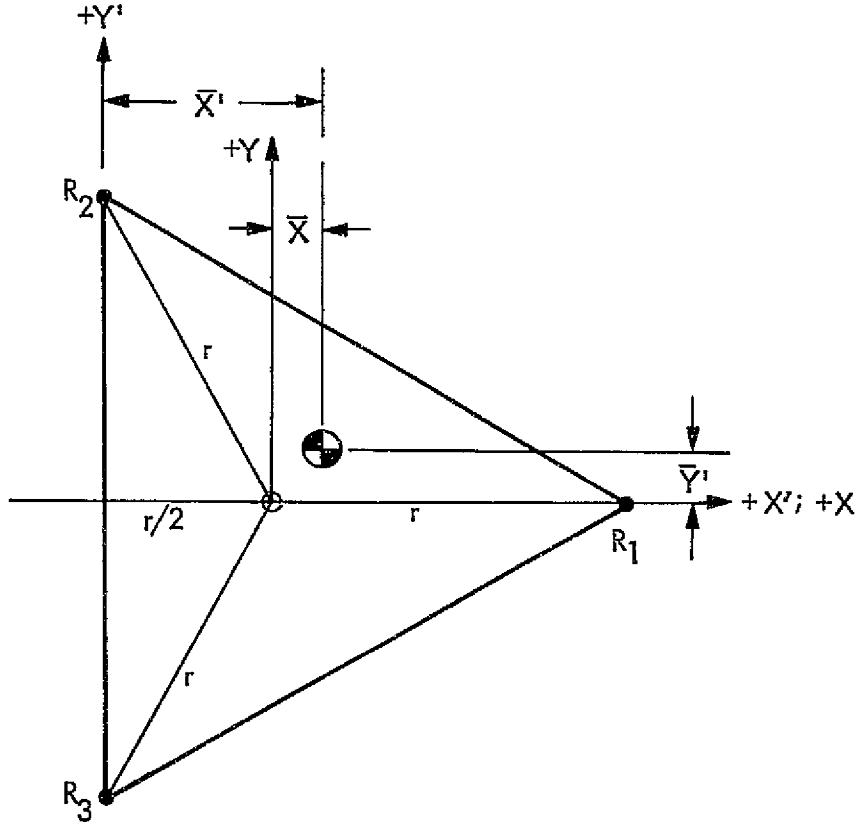

Fig. F.1. System Geometry and Notation

Thus, in the spacecraft system,

$$\bar{X} = \bar{X}' - \frac{r}{2} \quad : \quad \bar{Y} = \bar{Y}'$$

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The total weight is:

$$W = R_1 + R_2 + R_3 \tag{1}$$

From the moment equilibrium about the Y' axis

$$W\bar{X}' = R_1\left(\frac{3}{2}\,r\right)$$

$$\bar{X}' = \frac{3}{2}\,\frac{R_1}{W}\,r \tag{2}$$

From the moment equilibrium about the X' axis,

$$W\bar{Y}' = \frac{3}{2}\,\left(R_2 - R_3\right)r$$

$$\bar{Y}' = \frac{3}{2}\left(\frac{R_2 - R_3}{W}\right)r \tag{3}$$

2. Error analysis based on a single set of measurements.

In Eqs. (1) to (3), the reactions R_j are taken to be actual values, without error. Let the load cell readouts be defined as

$$P_i = k_i R_j \tag{4}$$

where k_i is a nondimensional constant. Then using P_i in place of R_j in Eqs. (1) and (2), and expressing errors in differential notation leads to

$$d\bar{x}' = r\,\frac{\partial\bar{x}'}{\partial r}\,\frac{dr}{r} + P_1\,\frac{\partial\bar{x}'}{\partial P_1}\,\frac{dP_1}{P_1} + W\,\frac{\partial\bar{x}'}{\partial W}\,\frac{dW}{W}$$

where

$$dW = P_1\,\frac{\partial W}{\partial P_1}\,\frac{dP_1}{P_1} + P_2\,\frac{\partial W}{\partial P_2}\,\frac{dP_2}{P_2} + P_3\,\frac{\partial W}{\partial P_3}\,\frac{dP_3}{P_3}$$

Then

$$d\overline{x}' = \overline{x}'\ \frac{dr}{r} + \overline{x}'\ \frac{dP_1}{P_1} - \overline{x}'\left(\frac{P_1}{W}\frac{dP_1}{P_1} + \frac{P_2}{W}\frac{dP_2}{P_2} + \frac{P_3}{W}\frac{dP_3}{P_3}\right)$$

As finite differences, considering that ΔP_i may be either positive or negative, the "worst case" gives

$$(\Delta\overline{x}')_{MAX} = \overline{x}'\left\{\frac{\Delta r}{r} + \left(1 - \frac{P_1}{W}\right)\frac{\Delta P_1}{P_1} + \frac{P_2}{W}\frac{\Delta P_2}{P_2} + \frac{P_3}{W}\frac{\Delta P_3}{P_3}\right\} \tag{5}$$

Similarly, it can be shown that

$$(\Delta\overline{Y}')_{MAX} = \overline{Y}'\left(\frac{\Delta r}{r} + \frac{\Delta P_1}{P_1} + \frac{\Delta P_2}{P_2} + \frac{\Delta P_3}{P_3}\right) + \frac{\sqrt{3}}{2}\, r\left(\frac{P_2}{W}\frac{\Delta P_2}{P_2} + \frac{P_3}{W}\frac{\Delta P_3}{P_3}\right) \tag{6}$$

If it is assumed that

$$P_i \approx P_j = P; \qquad \Delta P_i \approx \Delta P_j = \Delta P$$

$$\overline{x}' \approx \frac{r}{2}; \qquad \overline{Y}' << \frac{r\sqrt{3}}{2}$$

then Eqs. (5) and (6) become

$$(\Delta\overline{x}')_{MAX} \approx \frac{1}{2}\,\Delta r + \frac{2}{3}\,\frac{\Delta P}{P}\, r \tag{7}$$

$$(\Delta\overline{Y}')_{MAX} \approx \frac{\sqrt{3}}{3}\,\frac{\Delta P}{P}\, r \tag{8}$$

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3. Examination of $\Delta P/P$.

The k_i in Eq. (4) are nondimensional constants, and can be written

$$k_i = E_i\ S_i\ V_i \tag{9}$$

where V_i is the voltage readout from the load cell circuit, S_i is the basic sensitivity of the load cell in lbs/volt per volt, and E_i is the voltage from the bridge power supply. Dropping subscripts, we have, in general

$$P = ESVR$$

$$\frac{dP}{P} = \frac{E}{P}\frac{\partial P}{\partial E}\ \frac{dE}{E} + \frac{S}{P}\frac{\partial P}{\partial S}\frac{dS}{S} + \frac{V}{P}\frac{\partial P}{\partial V}\frac{dV}{V} + \frac{R}{P}\frac{\partial P}{\partial R}\frac{dR}{R} \tag{10}$$

Since, by definition, R is the actual load $\therefore \frac{dR}{R} = 0$

$$\frac{\Delta P}{P} = \frac{\Delta E}{E} + \frac{\Delta S}{S} + \frac{\Delta V}{V} \tag{11}$$

Regulated D.C. power supplies are available with a voltage stability $\Delta E/E$ of 0.005% over the test period. If a digital voltmeter is used for readout, there is an error $(\Delta V/V)_1$ of plus-or-minus one digit in the last place during (1) bridge balancing, and (2) during load readout. Considering that these errors can occur during load cell calibration and again during weight-and-C.G. measurement, the worst-case error where the calibration is relied upon is

$$\left(\frac{\Delta V}{V}\right)_1 = \frac{4 \times \Delta P}{P} = \frac{0.4}{P}$$

when readout to the nearest tenth-pound is provided. The other part of error $(\Delta V/V)_2$ relates to the reproducibility of the voltmeter from one measurement to another, rather than to the absolute calibration in volts. For the purposes of this analysis, the reproducibility is assumed to be 0.02% as an upper limit. Thus

$$\frac{\Delta P}{P} = 0.00005 + \frac{\Delta S}{S} + 0.0002 + \frac{0.4}{P}$$

Assume $P = 500$ lbs, then

$$\frac{\Delta P}{P} = 0.00105 + \frac{\Delta S}{S} \tag{12}$$

The basic sensitivity, S, can be deduced from the relation

$$S = \frac{C\Omega}{EVW} \tag{13}$$

where

Ω = resistance in ohms of a shunt calibration resistor across one leg of the load cell bridge

C = "equivalence factor," lbs/ohms

W = calibration weight, lbs

To clarify Eq. (13) it is noted that a standard calibration procedure is as follows:

(1) balance the bridge with no load

(2) load the cell with a calibration weight, W, near the rated capacity of the cell

(3) adjust the sensitivity of the readout box until the digital display coincides with the weight, W.

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(4) unload, and check zero.

(5) repeat these steps until zero load and calibration load readouts agree with actual loading

(6) insert shunt resistor and note "equivalent load"

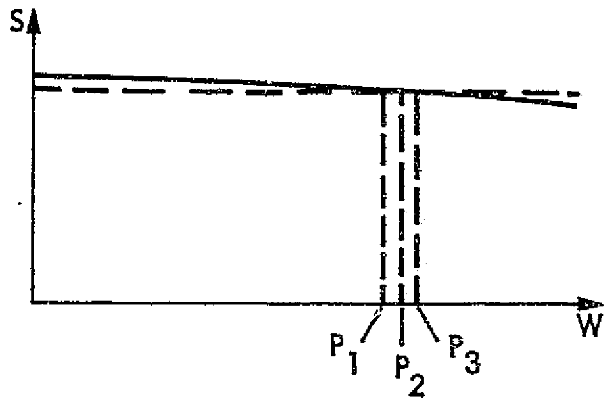
(7) proceed with calibration in load increments to obtain a check of linearity.

If the readout, P, is nonlinear with calibration weight, W, the calibration curve may be used in lieu of a constant sensitivity, S. All that is required for accurate measurement is that the calibration curve is reproducible.

An error analysis of S, based on Eq. (13) is not presented here, as it will be shown that it is of no practical concern in C.G. determination, although it is for the weight determination.

4. <u>A rudimentary C.G. measurement scheme.</u>

Assume that the test fixture uses only one load cell, with inert elements at the other two apices of the equilateral triangle. Assume that three separate measurements are made, with elements of the fixture rotated so that the load cell is subjected, successively, to the reactions R_1, R_2 and R_3. Furthermore, assume that the load cell sensitivity, S, is not independent of load, as depicted in the sketch below.



Then, with $P_1 \approx P_2 \approx P_3, \frac{\Delta S}{S} \approx 0$ and Eq. (12) becomes

$$\frac{\Delta P}{P} \approx 0.00105$$

Experience with load cell calibrations shows that nonlinearities are very small, and that the assumptions here are valid within measurement demands.

Let us now assume that $r = 24$ in., $\Delta r = .004$ in. Then, from Eqs. (7) and (8)

$$(\Delta \overline{X}')_{MAX} = 0.021 \text{ in.}$$

$$(\Delta \overline{Y}')_{MAX} = 0.015 \text{ in.}$$

$$\Delta r'_{MAX} = \sqrt{(\Delta \overline{X}')^2_{MAX} + (\Delta \overline{Y}')^2_{MAX}} = 0.026 \text{ in.}$$

This is a very good accuracy.

5. <u>The implemented system.</u>

The system implemented will use three load cells, instead of one. Here

$$P_{ij} = k_i P_j \tag{14}$$

where the subscript, i, refers to a particular load cell and the subscript, j, refers to a particular reaction. By defining the average C.G. coordinates as

$$(\overline{X}')_{AVE} = \frac{1}{3} (\overline{X}'_1 + \overline{X}'_2 + \overline{X}'_3)$$
$$(\overline{Y}')_{AVE} = \frac{1}{3} (\overline{Y}'_1 + \overline{Y}'_2 + \overline{Y}'_3) \tag{15}$$

a differential analysis reveals that the maximum errors are the same as those given by Eqs. (7) and (8).

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6. Other options.

The sections above discussed only one option in the assessment of errors in C.G. determination. This option entailed the use of instrumentation giving load cell readout to the nearest tenth-pound and a test procedure of three sets of measurements such that each of the three load cells measures each of the three spacecraft reactions. Three other options are presented here for an absolute calibration $\frac{\Delta S}{S} = .002$

Case 1. One pound readout; 3 load cells with one set of measurements

Then Eq. (12) of the reference becomes

$$\frac{\Delta P}{P} = .00005 + .002 + .0002 + \frac{4}{800}$$

$$= .00725$$

From Eqs. (7) and (8)

$$\left(\Delta\bar{X}'\right)_{MAX} = .118 \text{ in.}$$

$$\left(\Delta\bar{Y}'\right)_{MAX} = .100 \text{ in.}$$

$$\left(\Delta r'\right)_{MAX} = .155 \text{ in.}$$

Case 2. Five-tenth pound readout; 3 load cells with 1 set of measurement.

$$\frac{\Delta P}{P} = .00005 + .002 + .0002 + .0025$$

$$= .00475$$

$$\left(\Delta\bar{X}'\right)_{MAX} = .078 \text{ in.}$$

$$\left(\Delta\bar{Y}'\right)_{MAX} = .0658 \text{ in.}$$

$$\left(\Delta r'\right)_{MAX} = .102 \text{ in.}$$

Case 3. One-tenth pound readout; 3 load cells with one set of measurements

$$\frac{\Delta P}{P} = .00005 + .002 + .0002 + .0005$$

$$= .00275$$

$$\left(\Delta\bar{X}'\right)_{MAX} = .046 \text{ in.}$$

$$\left(\Delta\bar{Y}'\right)_{MAX} = .038 \text{ in.}$$

$$\left(\Delta r'\right)_{MAX} = .060 \text{ in.}$$

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Appendix G. STATISTICAL BEST FIT

SUMMARY

Weight and center of gravity measurement technique has been established as discussed in Sec. 2 of this report. Six locations of the apparent C.G. of the spacecraft are determined by making three 120^{o} load cell rotations and one 180^{o} spacecraft rotations to eliminate systematic errors. The present analysis finds two best fit equilateral triangles to the six points in a least square sense in order to improve the C.G. accuracy and give an error estimate.

The centroid of the two equilateral triangles is taken as a "better" C.G. and the standard deviation is calculated.

ANALYSIS

This analysis is aimed at minimizing instrumentation errors. Sec. 2 of this report shows that for each spacecraft rotation the apparent C.G. locations for three 120^{o} load cell rotations should form an equilateral triangle in absence of readout errors. The present analysis shows that the mean value is actually a best fit. The standard derivation σ for the obtainable accuracy is determined by this analysis and indicates what percent of the errors can be attributed to instrumentation by statistical estimate.

Six data points (x_i, y_i), $i = 1, \ldots 6$ are given by Eqs. (1), (2), (3) of Sec. 2. By rotating a vector whose x,y components are a and b through two 120^{o} angles, the vertices of an equilateral triangle are obtained, located close to one set of three data points corresponding to one spacecraft position.

A second equal and parallel triangle is similarly formed, located close to the other set of three data points for a 180° spacecraft rotation.

If (a_i, b_i), $i = 1 \ldots\ldots 3$ are the components of the three vectors for a triangle, and using the relationship

$$x' = x \cos \theta - y \sin \theta$$

$$y' = y \cos \theta - x \sin \theta$$

for angular rotation of the vector $\left[x, y\right]$ through θ, we obtain

$$a_1 = a \ , \ b_1 = b$$

$$a_2 = -\frac{1}{2}(a + b\sqrt{3}),\ b_2 = \frac{1}{2}(\sqrt{3}\,a - b)$$

$$a_3 = -\frac{1}{2}(a - b\sqrt{3}),\ b_3 = -\frac{1}{2}(\sqrt{3}\,a + b)$$

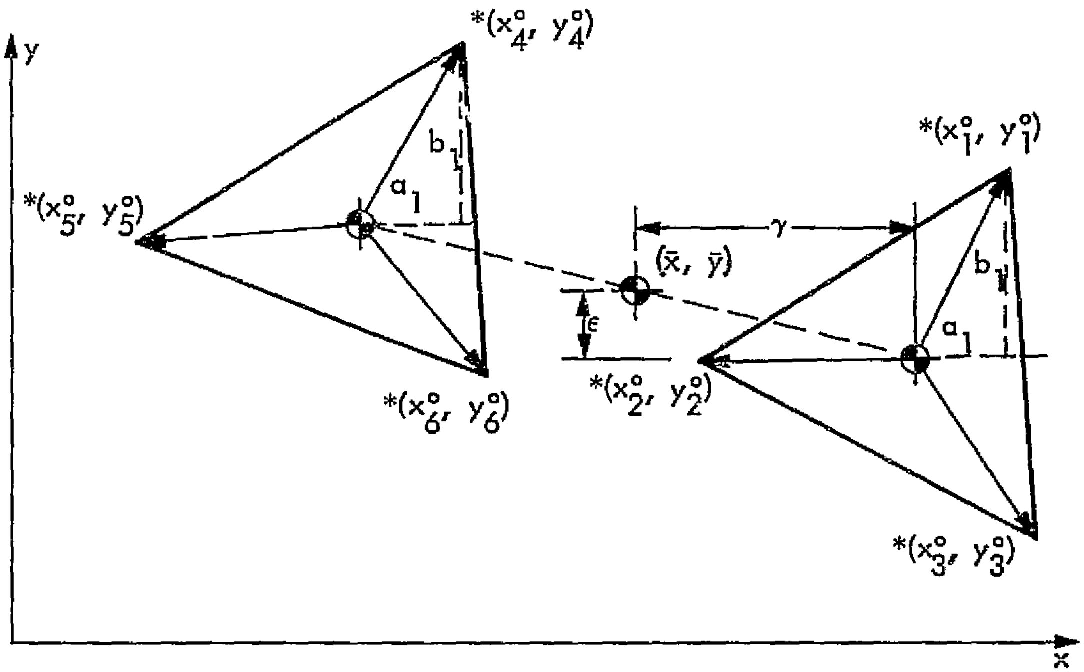

Fig. G.1. Best Fit Triangles

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If $(\overline{x},\overline{y})$ is the new C.G. and γ,ε are the vector components from this C.G. to the centroid of either triangle (see Fig. G.1) the coordinates of the vertices of one triangle can be written as $(\overline{x} + \gamma + a_i, \overline{y} - \varepsilon + b_i)$ where $i = 1, \ldots 3$ and $(\overline{x} - \gamma + a_i, \overline{y} + \varepsilon + b_i)$ where $i = 1, \ldots 3$ for the other. Let S^2 be the mean square error.

$$S^2 = \sum_{i=1}^{3}\left[x^o{}_i - (\overline{x} + \gamma + a_i)\right]^2 + \sum_{i=1}^{3}\left[y^o{}_i - (\overline{y} - \varepsilon + b_i)\right]^2$$

$$+ \sum_{i=1}^{3}\left[x^o{}_{i+3} - (\overline{x} - \gamma + a_i)\right]^2 + \sum_{i=1}^{3}\left[y^o{}_{i+3} - (\overline{y} + \varepsilon + b_i)\right]^2 \qquad (1)$$

The mean square S^2 is minimized by equating the partials $\partial S^2/\partial \overline{x}$, $\partial S^2/\partial \overline{y}$ $\partial S^2/\partial \gamma$, $\partial S^2/\partial \varepsilon$, $\partial S^2/\partial a$ and $\partial S^2/\partial b$ to zero. This set of equations is solved to give

$$\overline{x} = \frac{1}{6}\sum_{i=1}^{6} x^o{}_i, \quad \overline{y} = \frac{1}{6}\sum_{i=1}^{6} y^o{}_i \qquad (2)$$

$$\gamma = \frac{1}{6}\left[\sum_{i=1}^{3} x^o{}_i - \sum_{i=4}^{6} x^o{}_i\right], \quad \varepsilon = \frac{1}{6}\left[\sum_{i=1}^{3} y^o{}_i - \sum_{i=4}^{6} y^o{}_i\right] \qquad (3)$$

$$a = \frac{1}{12}\left[3(x^o{}_1 + x^o{}_4) - \sum_{i=1}^{6} x^o{}_i + \sqrt{3}\,(y^o{}_2 - y^o{}_3 + y^o{}_5 - y^o{}_6)\right] \qquad (4)$$

$$b = \frac{1}{12}\left[3(y^o{}_1 + y^o{}_4) - \sum_{i=1}^{6} y^o{}_i - \sqrt{3}\,(x^o{}_2 - x^o{}_3 + x^o{}_5 - x^o{}_6)\right] \qquad (5)$$

It is seen that $(\overline{x},\overline{y})$ is the mean of the data points. The standard deviation σ due to errors made in the measurements can now be estimated from Eq. (1). Noting that there are n=6 points, an unbiased estimate of the

standard deviation is obtained by dividing S^2 by $n - 1 = 5$.

$$\sigma = \sqrt{\frac{S^2}{5}} \qquad \text{(vector C.G.)} \qquad (6)$$

Standard Deviation σ_W for Weight

The values W_1, W_2 and W_3 refer to the weight of spacecraft and tare for one spacecraft position and W_4, W_5 and W_6 for a 180° rotation of the spacecraft. Recalling Eqs. (4) and (7) in Sec. 2, the standard deviation σ_W for the total weight can be calculated from

$$\sigma_W = \left[\frac{(W_1 - W)^2 + \ldots + (W_6 - W)^2}{5} + \sigma^2_{CAL}\right]^{1/2} \qquad (7)$$

where W is the mean value of the W_i's and σ_{CAL} is the standard deviation from the calibration curve of the load cells.

Standard Deviation for Spacecraft

Given tare data W_T, $\bar{X}_T$, $\bar{Y}_T$, σ_i and σ_{W_T} we calculate $\bar{X}_{S/C}$, $\bar{Y}_{S/C}$ and $W_{S/C}$ from Eqs. (8), (9) and (10) in Sec. 2, and also the standard deviations for:

(a) Spacecraft C.G.

$$\sigma_{S/C} = \frac{W}{W_{S/C}} \cdot \sigma \qquad (8)$$

b) Spacecraft Weight

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$$\sigma_{W_{S/C}} = \sqrt{\sigma^2_W + \sigma^2_{W_T}} \qquad (9)$$

A sample output and the program listing are in the Appendix E and A, respectively.

Appendix H. THE WEIGHT AND THE CENTER OF GRAVITY ERROR ESTIMATES

1. SUMMARY

It is noted that the effect of the spacecraft C.G. offset (from the center line of the load cell assembly) on the spacecraft C.G. error estimate is eliminated by taking two sets of measurements with the spacecraft rotated 180° between measurements. The obtainable accuracy on the spacecraft C.G. from the Voyager spacecraft weight and C.G. measurement fixture, with more realistic weight estimates and a more conservative tare C.G. location error estimate than Sec. 2, is estimated to be ±0.017 in. And, the corresponding accuracy in the spacecraft weight is estimated to be ±4.5 lbs.

2. ANALYSIS

From Eqs. (5) and (6) of Appendix F.

$$(\Delta\bar{x}')_{MAX} = \bar{x}'\left[\frac{\Delta r}{r} + \left(1 - \frac{P_1}{W}\right)\frac{\Delta P_1}{P_1} + \frac{P_2}{W}\frac{\Delta P_2}{P_2} + \frac{P_3}{W}\frac{\Delta P_3}{P_3}\right] \tag{5}$$

$$(\Delta\bar{y}')_{MAX} = \bar{y}'\left[\frac{\Delta r}{r} + \frac{\Delta P_1}{P_1} + \frac{\Delta P_2}{P_2} + \frac{\Delta P_3}{P_3}\right] + \frac{r\sqrt{3}}{2}\left[\frac{P_2}{W}\frac{\Delta P_2}{P_2} + \frac{P_3}{W}\frac{\Delta P_3}{P_3}\right] \tag{6}$$

where $P_1 + P_2 + P_3 = W$

These equations are simplified by assuming that

$$P_i \approx P_j = P; \ \bar{x}' \approx \frac{r}{2} \text{ and } \bar{y}' << \frac{r\sqrt{3}}{2}$$

Furthermore, $\Delta P_i \approx \Delta P_j = \Delta P$, since the three load cells must have the same bound. Therefore,

$$(\Delta\bar{x}')_{MAX} \approx \frac{\Delta r}{2} + \frac{2}{3}\frac{\Delta P}{P}r \tag{7}$$

$$(\Delta\bar{y}')_{MAX} \approx \frac{\sqrt{3}}{3}\frac{\Delta P}{P}r \tag{8}$$

The assumptions above can be relaxed with no change in Eqs. (7) and (8) other than $3P \rightarrow W$. The two sets of measurements taken with the spacecraft (with C.G. offset of $\overline{x},\overline{y}$) rotated 180° (not part of the derivation of Appendix F.) therefore eliminate the effect of C.G. offset of the spacecraft on the error analysis.

For, Position 1 $\quad \overline{x}'_1 = \frac{r}{2} + \overline{x}$

$\overline{y}'_1 = \overline{y}$

Position 2 (180° from 1) $\quad \overline{x}'_2 = \frac{r}{2} - \overline{x}$

$\overline{y}'_2 = -\overline{y}$

From Eq. (5) above

$$\Delta\overline{x}'_1 \leq \overline{x}'_1 \frac{\Delta r}{r} + \left(\frac{W\overline{x}'_1}{P_1} - \overline{x}'_1\right) \frac{\Delta P_1}{W} + \overline{x}'_1 \frac{\Delta P_2}{W} + \overline{x}'_1 \frac{\Delta P_3}{W}$$

and from Eq. (2) of Appendix F.

$$\overline{x}'_1 = \frac{3}{2} \frac{P_1}{W} r$$

Therefore,

$$\Delta\overline{x}'_1 \leq \overline{x}'_1 \frac{\Delta r}{r} + \left(\frac{3}{2} r - \overline{x}'_1\right) \frac{\Delta P_1}{W} + \overline{x}'_1 \frac{\Delta P_2}{W} + \overline{x}'_1 \frac{\Delta P_3}{W}$$

or

$$\Delta\overline{x}'_1 \leq \overline{x}'_1 \frac{\Delta r}{r} + \left(\frac{3}{2} r + \overline{x}'_1\right) \frac{\Delta P}{W}$$

Similarly,

$$\Delta\overline{x}'_2 \leq \overline{x}'_2 \frac{\Delta r}{r} + \left(\frac{3}{2} r + \overline{x}'_2\right) \frac{\Delta P}{W}$$

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Then

$$\Delta\overline{x}' = \frac{\Delta\overline{x}'_1 + \Delta\overline{x}'_2}{2} \leq \frac{\overline{x}'_1 + \overline{x}'_2}{2}\,\frac{\Delta r}{r} + \left(\frac{3}{2}\,r + \frac{\overline{x}'_1 + \overline{x}'_2}{2}\right)\frac{\Delta P}{W}$$

Therefore

$$\Delta\overline{x}' \leq \frac{\Delta r}{2} + 2r\,\frac{\Delta P}{W} \qquad (11a)$$

or

$$\Delta\overline{x}' \leq \frac{\Delta r}{2} + \frac{2}{3}\,\frac{\Delta P}{P}\,r$$ which is the same as Eq. (7) with $W \approx 3P$

From Eq. (6) above

$$\Delta\overline{y}'_1 \leq \overline{y}'_1\left(\frac{\Delta r}{r} + \frac{\Delta W}{W}\right) + \frac{r\sqrt{3}}{2}\left(2\,\frac{\Delta P}{W}\right)$$

or

$$\Delta\overline{y}'_1 \leq \overline{y}'_1\left(\frac{\Delta r}{r} + \frac{\Delta W}{W}\right) + r\sqrt{3}\,\frac{\Delta P}{W}$$

Similarly

$$\Delta\overline{y}'_2 \leq \overline{y}'_2\left(\frac{\Delta r}{r} + \frac{\Delta W}{W}\right) + r\sqrt{3}\,\frac{\Delta P}{W}$$

Then

$$\Delta\overline{y}' = \frac{\Delta\overline{y}'_1 + \Delta\overline{y}'_2}{2} \leq \frac{\overline{y}'_1 + \overline{y}'_2}{2}\left(\frac{\Delta r}{r} + \frac{\Delta W}{W}\right) + r\sqrt{3}\,\frac{\Delta P}{W}$$

Therefore

$$\Delta\overline{y}' \leq r\sqrt{3}\,\frac{\Delta P}{W}$$

or

$$\Delta\overline{y}' \leq \frac{\sqrt{3}}{3}\,\frac{\Delta P}{P}\,r$$ which is the same as Eq. (8) with $W \approx 3P$

3. ERROR ESTIMATE

a) C.G. location of spacecraft with Tare, and C.G. location of spacecraft. From Eqs. (11) and (12) where 3P is replaced by W of Sec. 2

$$\Delta\bar{X} \leqslant \frac{\Delta r}{2} + 2r\frac{\Delta P}{W} + \Delta x_o$$

$$\Delta Y \leqslant r\sqrt{3}\,\frac{\Delta P}{W} + \Delta y_o$$

Where Δx_o, Δy_o are concentricity error of the spacecraft longitudinal axis relative to the vertical reference axis of the weight and C.G. fixture as mentioned in Appendix F.

Let

$W_{S/C} = 1650$ lbs, $W_T = 300$ lbs

$r = 9.50$ in. $\Delta r = 0.002$ in.

$\Delta x_o = \Delta y_o = 0.0035$ in.

and $\Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3$

The readout error ΔP is a random error consisting of three parts, ΔP_1, ΔP_2 and ΔP_3 where

i. ΔP_1 is that part of the error due to the readout drifting during calibration. Its value has been determined to be 0.10 lb from the load cell calibration curves over the range of $0 \rightarrow 960$ lbs. for all three load cells. Only one reading among a total of 117 gives a $\Delta P_1 > 0.10$ lbs. and was discarded as an anomaly.

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ii. ΔP_2 is that part of the error due to the readout resolution. Its value has been chosen to be 0.1 lbs.

iii. ΔP_3 is that part of the error due to the readout fluctuation on account of the test site environment which includes ground vibration, wind draft, etc. Its value was not known so a range from $0 \rightarrow 0.50$ lbs was assumed.

Then the C.G. error was estimated giving the following table:

ΔP_3 (lb)	ΔP (lb)	$\Delta\bar{\bar{X}}$ (in.)	$\Delta\bar{\bar{Y}}$ (in.)	$\Delta CG_{S/C + T}$ (in.)	$\Delta\bar{X}_{S/C}$* (in.)	$\Delta\bar{Y}_{S/C}$* (in.)	$\Delta CG_{S/C}$ (in.)
0	0.20	0.0065	0.0052	0.008	0.0077	0.0062	0.010
0.10	0.30	0.0074	0.0060	0.010	0.0088	0.0071	0.011
0.20	0.40	0.0084	0.0069	0.011	0.0099	0.0082	0.013
0.30	0.50	0.0094	0.0077	0.012	0.0111	0.0091	0.014
0.50	0.70	0.0113	0.0094	0.015	0.0134	0.0111	0.018

*Calculated from Eqs. (13) and (14) of Sec. 2.

b) The error of weight measurement has been estimated by the following equation

$$\frac{\Delta W}{W} = \frac{\Delta\lambda}{\lambda} + \frac{\Delta P}{P}$$

or

$$\Delta W = \frac{\Delta\lambda}{\lambda} W + 3\Delta P$$

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where $W \approx 3P$

and $\frac{\Delta\lambda}{\lambda}$ is the load cell calibration accuracy, assumed to be 0.1%.

Therefore, for $W_{S/C}$ = 1650 lb, W_T = 300 lb., the errors of weight measurement are:

ΔP_3 (lb)	ΔP (lb)	$\Delta W_{S/C + Tare}$ (lb)	ΔW_{Tare} (lb)	$\Delta W_{net\ S/C}$ (lb)
0	.20	2.55	0.90	2.70
.10	.30	2.85	1.20	3.09
.20	.40	3.15	1.50	3.49
.30	.50	3.45	1.80	3.89
.50	.70	4.05	2.40	4.71

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